ISSN:2584-2706 DOI: https://doi.org/10.67317/IJMSRT26SEP050

A Simplex-Algorithm Approach to Optimal Product-Mix and Profit Maximisation in Sachet and Bottled Water Production: A Delta State Case Study

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Abstract
This paper applies the simplex algorithm of linear programming to the product-mix decision of an anonymised pure-water bottling factory in Agbor, Delta State, which produces three product lines [sachet water (bagged, 20 × 50cl sachets per bag), 50cl bottled water (24 bottles per carton) and 75cl can water (24 cans per carton)] from a shared filling-and-sealing line. Per-unit profit contributions (short-run contribution margins, before fixed overhead) and resource requirements (treated water, machine time, and a packaging working-capital budget) are derived explicitly from estimated unit prices, material costs and processing times, and combined with market-survey demand ceilings for each product to formulate a three-variable, six-constraint linear program. Starting from the all-slack basic feasible solution, the simplex method is solved to optimality in three non-degenerate pivot iterations, and the resulting basis is confirmed unique. The optimal continuous solution recommends approximately 614 bags of sachet water, 150 cartons of 50cl bottled water and 35 cartons of 75cl can water per week, for a maximum weekly contribution margin of ₦97,225; a feasible integer plan of 608, 150 and 35 units respectively attains ₦97,012, within 0.2% of the continuous optimum. Only the treated-water supply, the packaging budget, and the bottled-water demand ceiling bind at the optimum; the shadow price of the binding demand ceiling (₦221.58 per additional carton) is far larger than that of either binding resource constraint, and right-hand-side ranging shows this shadow price holds only while the ceiling is relaxed up to 220.46 cartons per week, beyond which can-water production is crowded out entirely and the basis changes. The paper illustrates, on a small and fully worked application, both the mechanics of the simplex algorithm and the economic information (optimal quantities, shadow prices, and their valid ranges) that a completed tableau yields.

Keywords:
Linear Programming; Profit Maximisation; Shadow Price; Water Bottling Industry; Nigeria.

1. Introduction

Small and medium-sized Nigerian manufacturers frequently produce more than one stock-keeping unit from a shared, capacity-constrained production line, and must decide, week to week, how much of each product to make when the profit margins, resource requirements and achievable sales volumes differ across products. This is the classical product-mix problem of operations research, and it is naturally formulated as a linear program: maximise total profit contribution subject to the resource and market limits that bound how much of each product can actually be produced and sold. The simplex algorithm, introduced by George Dantzig in 1947 and still the workhorse method taught for this class of problem, solves such a linear program by moving systematically from one vertex (basic feasible solution) of the feasible region to an adjacent, more profitable vertex, until no further improvement is possible (Taha, 2017; Hillier & Lieberman, 2015).

This paper applies the simplex method to the product-mix decision of a pure-water bottling factory operating in Agbor, Delta State; the firm's identity is withheld for commercial confidentiality, following the convention used in comparable Nigerian case studies (see Section 2). The firm produces three distinct product lines from a single, shared filling-and-sealing line: sachet water sold in bags, 50cl bottled water sold by the carton, and 75cl can water sold by the carton. Each product line differs in its selling price, its material and packaging cost, the machine time it consumes on the shared line, the raw treated water it requires, and the volume the market will currently absorb, the ingredients of a genuine, non-trivial product-mix problem rather than a case where the answer is obviously "produce only the most profitable item."

The paper's contribution is deliberately modest and pedagogical in scope: it is a basic, fully transparent application of the simplex algorithm rather than a methodological extension of it. Its value lies in three features that are not always present together in the applied Nigerian literature reviewed in Section 2: first, every profit and resource coefficient in the model is derived from a stated unit-economics calculation (Section 4) rather than presented as an unexplained given; second, the full simplex tableau sequence, not only the final answer, is shown, so that the entering-variable and leaving-variable logic at each pivot is auditable; and third, the completed tableau's dual (shadow-price) information is interpreted explicitly for its managerial content, going beyond the optimal quantities themselves to identify which constraint, if relaxed, would be most valuable to relax.

The remainder of the paper is organised as follows. Section 2 reviews related applications of linear programming and the simplex method to Nigerian production and product-mix problems. Section 3 sets out the general linear-programming formulation and the simplex algorithm. Section 4 describes the case-study factory and derives its profit coefficients and resource constraints explicitly. Section 5 reports the full simplex solution, an integer-feasible operational recommendation, and the economic interpretation of the shadow prices. Section 6 discusses the findings and their limits. Section 7 concludes.

2. Related Literature

Linear programming and the simplex method are standard tools for product-mix and profit-maximisation problems and are treated at length in general operations-research texts (Taha, 2017; Hillier & Lieberman, 2015). A substantial applied literature has used the simplex method on Nigerian manufacturing case studies. Hassan et al. (2020) applied a revised simplex approach to the product-mix problem of Amo Byng Nigeria Limited, an animal-feed manufacturer, and identified the profit-maximising combination of feed products given raw-material availability. Balogun and colleagues (2012) formulated a linear program for the Ilorin plant of the Nigerian Bottling Company to determine the profit-maximising mix of soft-drink products from shared bottling-line capacity, a problem structurally close to the present case study. Fagoyinbo et al. (2010) used linear programming to determine the maximum-profit product mix at Geepee Nigeria Limited, a plastics manufacturer, subject to raw-material constraints, and found that the unconstrained profit-ranking of products did not survive once resource limits were imposed, a pattern that recurs in the present case study. Raimi and colleagues (2017) applied the simplex method to bakery production planning at a polytechnic bakery in Owo, Ondo State, working, as here, with a small number of product variants sharing one production process.

Two observations from this literature motivate the present paper's emphasis. First, most published Nigerian case studies report the optimal product mix and the maximum profit but stop short of interpreting the dual (shadow-price) values that a completed simplex tableau also yields; these values identify which constraint is most valuable to relax and by how much, which is directly actionable management information that the optimal primal solution alone does not convey. Second, several studies report cost and price data as given without a stated derivation, which, as in the companion transportation-model study on the same firm's distribution network, limits how readily the model can be re-parameterised as input costs change. This paper accordingly derives its coefficients explicitly (Section 4) and reports the dual values alongside the optimal mix (Section 5.4).

ISSN:2584-2706
DOI: https://doi.org/10.67317/IJMSRT26SEP050

3. Linear-Programming Formulation and the Simplex Method

3.1 General Formulation

Let there be n products, indexed j = 1,…,n, with decision variable x_j denoting the quantity of product j to be produced in the planning period, and let c_j denote the profit contribution per unit of product j. Let there be m resource or market constraints, indexed i = 1,…,m, with a_{ij} the amount of resource i consumed per unit of product j and b_i the amount of resource i available. The product-mix linear program is:

Maximise $Z = \Sigma_j \, c_j \, x_j$

subject to $\Sigma_j \, a_{ij} \, x_j \leq b_i$ *for* $i = 1,\ldots,m$; $x_j \geq 0$ *for* $j = 1,\ldots,n$

Introducing a non-negative slack variable s_i for each constraint converts the m inequalities into equalities, $\Sigma_j \, a_{ij} \, x_j + s_i = b_i$, and the all-slack solution ($x_j = 0$ for all j, $s_i = b_i$ for all i) is an immediately available initial basic feasible solution, since every $b_i \geq 0$ in this problem.

3.2 The Simplex Algorithm

Starting from the initial all-slack tableau, the simplex algorithm proceeds as follows. (i) Compute, for every non-basic variable, the row $z_j - c_j$, where z_j is the profit-contribution-weighted sum of that variable's column entries over the current basis; for a maximisation problem in this form, the current solution is optimal once every $z_j - c_j \geq 0$. (ii) Otherwise, select the entering variable as the one with the most negative $z_j - c_j$ (Dantzig's rule). (iii) Perform the minimum-ratio test (dividing each basic variable's current value by its (strictly positive) coefficient in the entering variable's column) to determine the leaving variable, ensuring the new solution remains feasible. (iv) Pivot: divide the leaving variable's row by the pivot element, and eliminate the entering variable from every other row, producing a new tableau and a new, weakly more profitable basic feasible solution. Steps (i)–(iv) repeat until optimality is reached. Because each pivot strictly improves (or, in the case of a tie, does not worsen) the objective value and the number of basic feasible solutions of a bounded polytope is finite, the algorithm terminates in a finite number of iterations for the non-degenerate case encountered here.

At optimality, the $z_j - c_j$ value recorded under each slack variable's column is the shadow price (dual value) of the corresponding constraint: the marginal change in the optimal objective value per unit relaxation of that constraint's right-hand side, for small relaxations that do not change the optimal basis. A zero shadow price on a constraint whose slack is strictly positive in the optimal solution, and a strictly positive shadow price only on constraints whose slack is exactly zero, is guaranteed by the complementary-slackness property of linear programming duality (Taha, 2017; Hillier & Lieberman, 2015), and is used directly in Section 5.4 to interpret the case-study result.

Two further checks are routinely applied alongside the pivoting itself. First, degeneracy (a basic variable taking the value zero at some iteration) can, in principle, cause the algorithm to cycle without terminating; Bland's rule (always selecting the lowest-indexed eligible entering and leaving variable) is the standard safeguard where this is a risk (Taha, 2017). Second, once an optimal tableau is reached, checking whether any non-basic variable's reduced cost is exactly zero indicates the existence of an alternative optimal basis with the same objective value; if every non-basic reduced cost is strictly positive, the optimal solution is unique. Both checks are reported for the case study in Section 5.3.

4. Case-Study Model: Products, Profit Coefficients and Constraints

The factory studied packages three products from one shared filling-and-sealing line: x_1, bags of sachet water (20 × 50cl sachets per bag, 10 litres of treated water per bag before wastage); x_2, cartons of 50cl bottled water (24 bottles per carton, 12 litres per carton before wastage); and x_3, cartons of 75cl can water (24 cans per carton, 18 litres per carton before wastage). All figures below are simulated for illustrative purposes, calibrated to plausible unit economics for a mid-sized Nigerian sachet/bottled-water bottler in the current cost environment, and the firm's name is withheld for commercial confidentiality.

4.1 Profit Coefficients

Volume 4 / Issue 9 / September 2026 International Journal of Modern Science and Research Technology
ISSN:2584-2706 DOI: https://doi.org/10.67317/IJMSRT26SEP050

Profit per unit is derived as selling price less the sum of raw-water cost, packaging-material cost and direct-labour cost, with a 5% production wastage allowance applied to the raw-water requirement. A treated-water cost of ₦2.00/litre and a labour rate of ₦8.00 per machine-minute (covering wages and line overheads apportioned per minute of run time) are estimated throughout. Table 1 sets out the derivation for each product.

Table 1: Derivation of unit profit contribution (₦)

Component	x_1 Sachet bag	x_2 50cl carton	x_3 75cl carton
Water use incl. 5% wastage (L)	10.5	12.6	18.9
Raw-water cost (₦2.00/L)	21.00	25.20	37.80
Packaging-material cost	25.00	840.00	1,680.00
Machine time (min)	1.2	3.5	4.0
Labour cost (₦8.00/min)	9.60	28.00	32.00
Total unit cost	55.60	893.20	1,749.80
Estimated selling price	95.00	1,300.00	2,100.00
Unit profit contribution, c_j	39.40 ≈ 39	406.80 ≈ 407	350.20 ≈ 350

Notably, the can-water product carries the highest selling price but not the highest unit profit, because aluminium can packaging costs considerably more per unit than PET bottle packaging; this is what prevents the problem from collapsing trivially into "produce only the highest-margin item" and is representative of genuine product-mix trade-offs in packaged-goods manufacturing. It should also be noted explicitly that c_j, as derived in Table 1, is a short-run contribution margin (revenue less variable cost) and excludes fixed costs such as rent, machinery depreciation, insurance and administrative overhead. The objective function value maximised throughout this paper is accordingly a weekly contribution margin, not a final accounting profit figure; converting it to net accounting profit would require deducting the firm's weekly fixed costs, which are not modelled here since they do not vary with the product mix and therefore do not affect the optimisation.

4.2 Resource and Demand Constraints

Three resource constraints are modelled: weekly treated-water availability, limited by borehole yield and treatment-plant throughput; weekly machine time on the shared filling-and-sealing line; and a weekly packaging-material working-capital budget, reflecting the cash-flow limit within which a small firm can pre-purchase nylon, PET preforms and aluminium cans ahead of sales revenue. In addition, each product faces a market-demand ceiling, reflecting current distributor order patterns and shelf penetration, beyond which additional output could not be sold in the coming week.

Table 2: Resource requirements per unit and weekly availability

Constraint	x_1 (per bag)	x_2 (per carton)	x_3 (per carton)	Available/week
Treated water (L)	10.5	12.6	18.9	9,000 L
Machine time (min)	1.2	3.5	4.0	1,500 min
Packaging budget (₦)	25	840	1,680	₦200,000
Market demand ceiling				$x_1 \leq 900$, $x_2 \leq 150$, $x_3 \leq 80$

The resulting linear program is:

ISSN:2584-2706 DOI: https://doi.org/10.67317/IJMSRT26SEP050

$$\begin{aligned}
&\textit{Maximise}\ Z = 39x_1 + 407x_2 + 350x_3 \\
&\textit{subject to}\ 10.5x_1 + 12.6x_2 + 18.9x_3 \leq 9{,}000 \quad \textit{(treated water)} \\
&\qquad 1.2x_1 + 3.5x_2 + 4.0x_3 \leq 1{,}500 \quad \textit{(machine time)} \\
&\qquad 25x_1 + 840x_2 + 1{,}680x_3 \leq 200{,}000 \quad \textit{(packaging budget)} \\
&\qquad x_1 \leq 900,\ x_2 \leq 150,\ x_3 \leq 80 \quad \textit{(demand ceilings)} \\
&\qquad x_1, x_2, x_3 \geq 0
\end{aligned}$$

The demand ceilings are written as explicit constraints (rather than as variable bounds) so that their slack variables, and hence their shadow prices, appear directly alongside those of the three resource constraints in the simplex tableau, which is convenient for the comparison drawn in Section 5.4.

5. Simplex Solution and Results

Adding slack variables $s_1,\ldots,s_6$ to the treated-water, machine-time, packaging-budget, and the three demand-ceiling constraints respectively gives the standard-form model solved below. The initial basic feasible solution sets every product quantity to zero and every slack equal to its constraint's right-hand side.

5.1 Iteration 0 - Initial Tableau

Table 3: Initial simplex tableau (all-slack basis)

Basis	x_1	x_2	x_3	s_1	s_2	s_3	s_4	s_5	s_6	RHS
s_1	10.5	12.6	18.9	1	0	0	0	0	0	9,000
s_2	1.2	3.5	4.0	0	1	0	0	0	0	1,500
s_3	25	840	1,680	0	0	1	0	0	0	200,000
s_4	1	0	0	0	0	0	1	0	0	900
s_5	0	1	0	0	0	0	0	1	0	150
s_6	0	0	1	0	0	0	0	0	1	80
$z_j - c_j$	−39	−407	−350	0	0	0	0	0	0	Z = 0

Every non-basic $z_j - c_j$ is negative, so the initial solution (produce nothing) is not optimal. The most negative entry, −407 under x_2, identifies x_2 (50cl bottled water) as the entering variable. The minimum-ratio test (RHS ÷ x_2-column entry, over strictly positive entries) gives 9000/12.6 = 714.3, 1500/3.5 = 428.6, 200000/840 = 238.1, and 150/1 = 150; the smallest ratio, 150, occurs at row s_5 (the bottled-water demand ceiling), which therefore leaves the basis.

5.2 Iteration 1

Pivoting on the x_2 column / s_5 row brings x_2 = 150 into the basis. The updated tableau (values rounded to 3 d.p.) has basic variables s_1 = 7,110, s_2 = 975, s_3 = 74,000, s_4 = 900, x_2 = 150, s_6 = 80, at Z = ₦61,050, with reduced costs $z_j - c_j$ of −39 (x_1), 0 (x_2, now basic), −350 (x_3), and +407 under s_5 (confirming s_5's shadow price once it re-enters the discussion in Section 5.4). Since x_3 still shows a negative reduced cost, x_3 (can water) enters next. The ratio test over the x_3 column gives 7110/18.9 = 376.2, 975/4.0 = 243.75, 74000/1680 = 44.05, and 80/1 = 80; the minimum, 44.05, occurs at row s_3 (the packaging budget), which leaves the basis.

5.3 Iteration 2 and Final (Optimal) Tableau

Pivoting on the x_3 column / s_3 row brings x_3 = 44.048 into the basis, at Z = ₦76,466.67, with x_1 now the only remaining negative reduced cost (−33.792). x_1 (sachet water) therefore enters; the ratio test identifies row s_1 (treated water) as the leaving row, with ratio 6,277.5/10.219 ≈ 614.312. Pivoting on this element yields the final tableau, summarised in Tables 4a and 4b.

Table 4a: Final (optimal) basic variable values

Basic variable	x_1	s_2	x_3	s_4	x_2	s_6
Value	614.312	98.201	34.906	285.688	150.000	45.094

Table 4b: Shadow prices ($z_j - c_j$) of the six constraints at the optimum

Constraint	s_1 Water	s_2 Machine time	s_3 Budget	s_4 Sachet ceiling	s_5 Bottle ceiling	s_6 Can ceiling
Shadow price (₦)	3.307	0.000	0.171	0.000	221.583	0.000

Every reduced cost in the x_1, x_2, x_3 columns of the final tableau is now non-negative, so the solution is optimal: produce 614.312 bags of sachet water, 150.000 cartons of 50cl bottled water, and 34.906 cartons of 75cl can water per week, for a maximum weekly contribution margin of ₦97,225.29. This was independently corroborated by solving the underlying linear program directly with the HiGHS simplex solver, which returned an identical solution, confirming the manual pivoting sequence above. Two further checks noted in Section 3.2 were also carried out. Non-degeneracy: every basic variable at every one of the three iterations (Table 3 and the two intermediate tableaux of Sections 5.2–5.3) took a strictly positive value, so no degenerate pivot occurred and Bland's rule was not required. Uniqueness: in the final tableau, the only non-basic variables are s_1, s_3 and s_5 (Table 4b), and their reduced costs (3.307, 0.171 and 221.583) are all strictly positive; since no non-basic variable has a zero reduced cost, no alternative optimal basis exists with the same objective value, and the reported product mix is therefore the unique optimiser of the LP relaxation.

5.4 Shadow Prices and Their Interpretation

The $z_j - c_j$ values recorded under the slack columns of the final tableau (Table 4b) are the shadow prices of the six constraints, shown in Figure 1. Treated water (s_1) has a shadow price of ₦3.307 per litre: one additional litre of weekly treated-water capacity would raise maximum contribution margin by approximately ₦3.31. The packaging budget (s_3) has a shadow price of ₦0.171 per Naira: one additional Naira of weekly packaging working capital would raise contribution margin by about 17 kobo. This figure is a marginal, local rate of return (valid only over the range established in Section 5.5) and should not be read as a claim about the average return on packaging capital generally; it says only that the next Naira added to this particular constraint, at this particular operating point, is worth about 17 kobo. The bottled-water demand ceiling (s_5) has a shadow price of ₦221.583 per carton (one to two orders of magnitude larger than either binding resource constraint) meaning that securing one additional carton of sellable weekly demand for 50cl bottled water would be worth roughly ₦222 in additional contribution margin, far more than the value of a marginal litre of water or a marginal Naira of packaging budget, again subject to the valid range established next. Machine time (s_2), the sachet-water demand ceiling (s_4) and the can-water demand ceiling (s_6) all have zero shadow price, consistent with each having positive slack in the optimal solution (98.201 minutes, 285.688 bags and 45.094 cartons respectively) under the complementary-slackness property noted in Section 3.2: a constraint with room to spare contributes nothing at the margin, however much or little of it remains unused.

ISSN:2584-2706 DOI: https://doi.org/10.67317/IJMSRT26SEP050

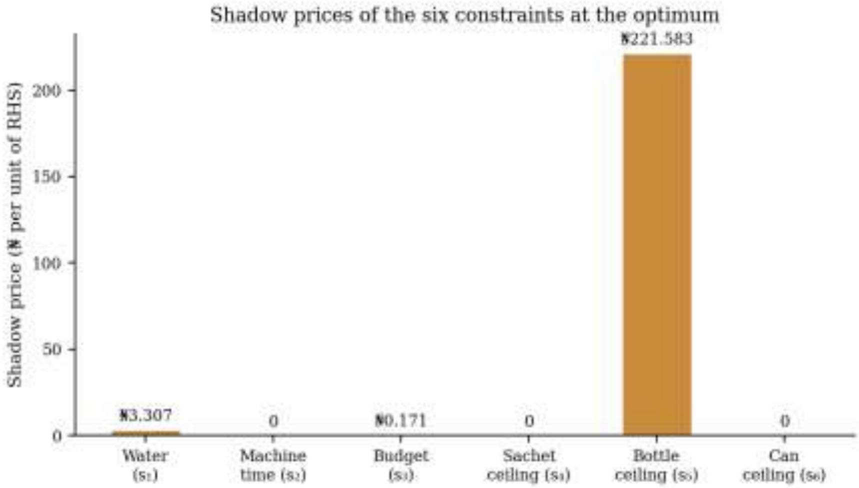

Figure 1: Shadow prices of the six constraints at the optimum

The managerial implication is direct and, on its face, counter-intuitive: although the factory is, in one sense, resource-constrained (it cannot produce unlimited quantities of any product), the single highest-value intervention available to this firm is not a resource expansion — not a bigger borehole, not a faster filling line, not a larger packaging budget — but market development for 50cl bottled water specifically, since that is the only demand ceiling currently binding, and it binds far more expensively, in shadow-price terms, than either of the two binding resource constraints. This is precisely the kind of conclusion that a completed simplex tableau makes visible but that the optimal quantities alone would not.

5.5 Right-Hand-Side Ranging: How Far Do the Shadow Prices Hold?

A shadow price is a marginal, local statement: it holds only while the corresponding right-hand side is varied within a specific range that leaves the optimal basis unchanged. This range was computed directly from the basis inverse of the final tableau, following standard linear-programming sensitivity theory (Taha, 2017), for each of the three binding constraints; the results are reported in Table 5.

Table 5: Valid right-hand-side range for each binding constraint's shadow price

Constraint	Current RHS	Valid range	Variable that would first bind beyond the range
Treated water (s_1)	9,000 L	2,722.5 – 9,879.9 L	Sachet-water production (x_1) at the upper end
Packaging budget (s_3)	₦200,000	₦142,928.6 – ₦273,728.6	Can-water production (x_3) at the lower end
Bottle-demand ceiling (s_5)	150 cartons	58.98 – 220.46 cartons	Can-water production (x_3), which reaches zero at the upper end

The bottled-water demand-ceiling result answers directly the question raised by the shadow-price finding of Section 5.4: the ₦221.583-per-carton value holds only while the ceiling is relaxed from its current 150 cartons up to 220.46 cartons per week. Over that range, every additional unit of achievable bottled-water demand displaces can-water production one-for-one at the margin, and at exactly 220.46 cartons, can-water production (x_3) reaches zero and leaves the basis entirely, beyond that point the factory would be producing only sachet and bottled water, the shadow price of a further relaxation would need to be recomputed from a new tableau, and the marginal value of additional bottled-water demand would very likely fall, since machine time (98.201 minutes of slack at the original optimum) would then be the next constraint to approach bindingness. This means the

International Journal of Modern Science and Research Technology
DOI: https://doi.org/10.67317/IJMSRT26SEP050

practical recommendation of Section 5.4 (prioritise market development for 50cl bottled water) is on firm ground for demand growth up to roughly 47% above the current ceiling, but should be re-evaluated, not simply extrapolated, beyond that point.

The treated-water and packaging-budget ranges in Table 5 are reported for completeness and to substantiate the local-validity caveat of Section 5.4: the packaging budget's modest shadow price of ₦0.171 per Naira, for instance, holds for budget increases up to ₦73,728.6 (a 36.9% increase), beyond which sachet-water production would already be constrained by the treated-water supply rather than by budget, again requiring the tableau to be re-solved rather than the existing shadow price to be extrapolated indefinitely.

5.6 Integer-Feasible Operational Recommendation

Because sachet bags and cartons are indivisible units, the continuous optimum of Section 5.3 (614.312 sachet bags, 34.906 can cartons) is not directly implementable and must be rounded. Fixing $x_2 = 150$ (its ceiling, which the shadow-price analysis above confirms should not be reduced) and enumerating exhaustively over the integer feasible region $x_1 \in \{0,\ldots,900\}$, $x_3 \in \{0,\ldots,80\}$ subject to the three resource constraints identifies 608 sachet bags and 35 can cartons as the contribution-margin-maximising integer combination, exhausting the packaging budget exactly (₦200,000 of ₦200,000) while leaving 64.5 litres of treated water and 105.4 minutes of machine time unused.

Table 6: Recommended integer weekly production plan

Product	Recommended quantity	Weekly contribution margin (₦)
x_1 – Sachet water (bags)	608	23,712
x_2 – 50cl bottled water (cartons)	150	61,050
x_3 – 75cl can water (cartons)	35	12,250
Total weekly contribution margin		97,012

This integer plan attains 99.78% of the continuous-relaxation optimum (₦97,012 versus ₦97,225.29), a gap small enough that, for a weekly operational decision of this size, exhaustive enumeration over the relevant integer range is an adequate substitute for a full integer-programming (branch-and-bound) solution; the latter is noted as a natural extension in Section 6.

6. Discussion

Four observations follow from the case study. First, the optimal mix uses all three products, but not in the same order as their raw selling-price ranking would suggest: the highest-priced product, can water, is not the most profitable per unit once its packaging cost is accounted for, and even the most profitable product, bottled water, is capped well below what the plant could physically produce because the market, not the factory, is the binding limit on that line. This mirrors the finding in Fagoyinbo et al. (2011), where the unconstrained profitability ranking of products did not survive the imposition of resource and market limits, and it is a useful corrective to the intuition that a firm should simply “make more of whatever sells for the highest price.”

Second, the shadow-price analysis of Section 5.4 illustrates why reporting the optimal quantities alone, as much of the applied literature reviewed in Section 2 does, leaves value on the table analytically as well as operationally: the same completed tableau that yields the optimal mix also ranks, in Naira terms, exactly which constraint is most worth relaxing. In this case study that ranking is unambiguous and somewhat surprising (market access for one specific product line dominates every physical resource constraint by more than an order of magnitude) which is precisely the kind of finding that would not be visible from the optimal solution vector alone.

Third, the right-hand-side ranging of Section 5.5 qualifies that finding usefully rather than undermining it: the demand-ceiling shadow price is not a claim that could be extrapolated indefinitely, and reporting its valid range (up to 220.46 cartons, a 47% increase over the current ceiling) turns a directional recommendation into a bounded, checkable one. The same ranging exercise also confirms, via the strictly positive reduced costs on every non-basic variable in the final tableau, that the reported product mix is the unique optimiser of the LP relaxation and not one of several tied alternatives, a

ISSN:2584-2706 DOI: https://doi.org/10.67317/IJMSRT26SEP050

check that is straightforward once the final tableau is available but is rarely reported in the comparable applied literature.

Fourth, as with the companion transportation-model study on this firm's distribution network, the coefficients used here are simulated but derived rather than estimated, and the same caveats apply: selling prices, unit costs, processing times and demand ceilings should be replaced with the firm's own records before any operational reliance on the specific quantities reported, and the model should be re-solved whenever a cost or price input changes materially, which the explicit derivation in Section 4 is designed to make straightforward. It is also worth restating plainly that the objective maximised throughout is a short-run contribution margin, not final accounting profit; a firm applying this model to its own data would still need to net out its weekly fixed costs separately to arrive at accounting profit, though those costs do not affect which product mix is optimal. The model is also, deliberately, a basic one: it assumes deterministic, single-period demand and cost data, treats the three demand ceilings as hard limits rather than as prices the firm could pay to relax (e.g., through advertising spend, whose own cost-effectiveness is not modelled here), and stops at a rounded integer recommendation rather than a full mixed-integer-programming solution. Extensions incorporating multi-period planning, stochastic demand, or an explicit advertising/market-development decision variable — which the shadow-price finding of Section 5.4 directly motivates — are natural next steps.

7. Conclusion and Recommendations

This paper applied the simplex algorithm to a basic three-product product-mix problem for an anonymised pure-water bottling factory in Agbor, Delta State, deriving every profit-contribution and resource coefficient explicitly from stated unit-economics assumptions rather than presenting them as given. Solved from the all-slack initial tableau through three non-degenerate pivot iterations, and confirmed unique via the strictly positive reduced costs of the final tableau, the optimal continuous product mix is approximately 614 bags of sachet water, 150 cartons of 50cl bottled water and 35 cartons of 75cl can water per week, for a maximum weekly contribution margin of ₦97,225; a feasible integer plan of 608, 150 and 35 units respectively attains ₦97,012, within 0.2% of the continuous optimum. The completed tableau's shadow prices show that only the treated-water supply, the packaging budget, and the bottled-water demand ceiling bind at the optimum, and that the demand ceiling's shadow price (₦221.58 per carton) dwarfs that of either binding resource constraint, identifying market development for 50cl bottled water, rather than any production-capacity expansion, as this firm's highest-value operational priority, valid, by right-hand-side ranging, for demand growth up to roughly 47% above the current ceiling.

Four recommendations follow. First, firms in this position should solve their product-mix decision as a linear program rather than by rule of thumb, since, as this case study shows, the price-based ranking of products does not necessarily match their contribution-maximising production ranking once resource and market constraints are imposed. Second, firms and researchers applying the simplex method to this class of problem should report and interpret the shadow prices of the final tableau, not only the optimal quantities, since the shadow prices identify which constraint is most valuable to relax and can materially change a firm's investment priorities, as illustrated directly in Section 5.4. Third, any reported shadow price should be accompanied by its valid right-hand-side range, as in Section 5.5, since a shadow price applied outside that range is not merely imprecise but can be actively misleading about where a firm's next unit of investment should go. Fourth, future work should extend the present deterministic, single-period, continuous-relaxation model to a full mixed-integer program, to multi-period planning that reflects seasonal demand variation, and to a formulation in which market-development spend is itself a decision variable, given the strong indication from the shadow-price analysis that demand expansion, not production expansion, is where marginal investment would be best directed for this firm.

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ISSN:2584-2706 DOI: https://doi.org/10.67317/IJMSRT26SEP050

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