Finite Element Computations of Blood Flow through Artery in the Presence of Mild Stenosis

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Abstract

The present paper deals with a computational study of blood flow through an artery in the presence of mild stenosis. Attempt has been made to analyse of velocity, pressure, momentum and various parameters of flow.Finite Element Method is used to handle the differential equations occurring during the study.The development and progression of cardiovascular diseases are major health concerns. The present study of blood circulation in artery with mild stenosis is for understanding and enhancing treatments. and mild stenoses alter normal blood flow characteristics, leading to serious health complications,abnormal blood flow patterns and high wall shear stresses can help in detecting cardiovascular diseases at an early stage.

Key words:
Mild stenoses, Cardiovascular disease, Shear stress

1.Introduction

In present age in India the increasing rate of cardiovascular diseases is a major challenge to the medical Scientists. Seeing this serious issue scientists have focused their attention towards this side. In the present study an effort is made to create a two-dimensional model of the shape of a non-uniform artery wall that has a restricted segment, using a segmented function, which includes an obstruction .. The blood flow in the body follows a rhythmic pressure gradient that imitates the heart's systolic and diastolic phases. because blood behaves like a non-Newtonian fluid in certain situations.

A lot of work has already done in this field.here we like to describe some important work and their contribution here.The Casson model for non-Newtonian fluids is used to account for the yield stress resulting from the formation of red blood cell aggregates at low shear rates.The Navier–Stokes equations, which describe incompressible and unsteady fluid flow, are expanded to include the non-Newtonian behavior of blood flow in radial coordinates. This is accomplished by including a temperature equation.To analyze the impact of stenosis over the flow, drug delivery agents such copper Cu and alumina nanoparticles with a concentration of about 0.03 are already studied.The concept of magnetohydrodynamics involves applying a magnetic field to blood flow in an artery, taking into account the Hall current, to guide magnetic drug carriers to a specific location within the bloodstream.

The flow of blood begins from a state of rest with zero velocity and certain initial conditions to simplify the mathematical modeling process. Along the axis, a zero radial gradient condition is applied to velocity and temperature, and no-slip conditions are applied at the arterial wall.Some researchers has done the excellent work in this field,we will like to explore their work which motivated us to work on this topic .Dabiri J, O at el (2013) did work on measuring ostrocard, swimming group flow. Katija k. (2014) studied quantitatively measuring in situ

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flows using a self contained under water velocimetry. Ruizla (2015) studied vertex enhanced propulsion. Collin at ei (2016) did work on stealth predation, the basis of ecological success. Whittlesey at el (2017) did work on schooling as a basis for vertical axis wind turbine farm design.

Breithburg at el (2018) studied ecosystem engineers in the pelagicrealm. J.c at el (2019) did work on Phenotypic plasticity in Juvenile Jellipish medusae facilitates effective fluid interaction. J.O, at el (2020) worked on a Lagrangian approach to identify vortex pinch of chaos 20. Rosinfield at el (2021) studied circulation generation and vortex ring formation by statics conic nozzles. Kratika & Dabril (2022) studied a viscosity enhanced mechanism for biogenic ocean mixing.

2. Mathematical Modeling and Governing Equations

This model treats blood flow as steady, laminar, and incompressible in a cylindrical tube, with a small axisymmetric narrowing due to mild stenosis It uses a perturbation approach valid when the stenosis is mild (typically height $\delta \ll$ radius R_0, or area reduction < ~50–60%).

3.Geometry of Mild Stenosis: The arterial radius in the stenosed region is modeled as:

$r(z) = R_0 [1 - \delta/R_0 \cdot f(z)]$ or can be written as $r(z) = R(z))$

where:

R_0 = normal artery radius

δ = maximum height/radial protrusion of the stenosis ($\delta \ll R_0$ for mild case)

$f(z)$ = axial shape function of the stenosis, usually Gaussian or cosine form for smooth stenosis. Common choices for $f(z)$:

$f(z) = \exp(-z^2 / (2\sigma^2))$ (σ controls length)

very common in mild stenosis $f(z) = (1/2) [1 + \cos(\pi z / L)]$ for $|z| \le L/2$, and $f(z) = 0$ otherwise

(L = length of stenosis)

The degree of stenosis is can also be expressed as percentage area reduction:

Percentage of stenosis = $(1 - (R\ min / R_0)^2) \times 100\%$

For mild stenosis: typically < 50% area reduction.

4.Governing Equations: Under mild stenosis approximation ($\delta/R_0 \ll 1$, slope of wall is small → lubrication-like or quasi-Poiseuille flow), the simplified momentum equation from Navier–Stokes in cylindrical coordinates, axial direction becomes:

$dp/dz = (1/r)\ d/dr\ [\ r\ \mu\ du/dr\ ]$ (radial viscous term dominant)

with no-slip boundary condition: $u(r(z), z) = 0$ at $r = r(z)$

Integrating twice gives the velocity profile parabolic-like, but radius varies slowly with z.

$u(r,z) = (1/(4\mu))\ (-dp/dz)\ [r(z)^2 - r^2]$

Volumetric flow rate Q is constant (continuity):

$Q = \int_0^{r(z)} 2\pi\ r\ u(r,z)\ dr = -\ (\pi\ r(z)^4\ /\ (8\mu))\ dp/dz$

→ rearranged (Poiseuille form modified for varying radius):

$dp/dz = -\ (8\mu\ Q)\ /\ (\pi\ r(z)^4\ )$

5.Pressure drop across mild stenosis

Integrate along the vessel length:

$\Delta p = \int dp = (8\mu\ Q\ /\ \pi) \int dz\ /\ r(z)^4$

For the cosine shape vary standard in mild stenosis models, after integration over the stenosis length:

$\Delta p \approx (8\mu\ Q\ L\ /\ (\pi\ R_0^4)) \times [1 + (3/2)(\delta/R_0)^2 +$ higher-order terms] . More precisely $\Delta p = (8\mu\ Q\ /\ \pi\ R_0^4) \times (L + (3\pi^2/8)\ (\delta^2\ /\ R_0^2)\ (L/2)$ The additional pressure drop due to stenosis beyond normal vessel scales approximately as:Δp stenosis $\propto (\delta\ /\ R_0)^2$.This is much smaller than Poiseuille's r^{-4} for severe cases — that's why mild stenosis causes only modest hemodynamic effect.For non-Newtonian blood Casson, Herschel-Bulkley, Power-law models very common extensions, the expressions become more involved, but the mild approximation still holds and adds a yield stress or power-law correction.Non-Newtonian Models for Blood flow.Blood is a non-Newtonian fluid primarily due to the presence of red blood cells (RBCs), which cause shear-thinning viscosity decreases with increasing shear rate) and a small yield stress. In large arteries with high shear rates (> 100 s^{-1}), blood often behaves approximately as Newtonian, but non-Newtonian effects become significant in smaller vessels, low-flow regions, or post-stenotic zones where recirculation and low shear occurs.

6.Momentum Equations

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Under mild stenosis approximation, the simplified momentum equation becomes:
$dp/dz = (1/r)\, d/dr\, [\, r\, \mu\, du/dr\,]$
Velocity profile:
$u(r,z) = (1/(4\mu))\, (-dp/dz)\, [r(z)^2 - r^2]$
This is a compact mathematical model used for mild stenosis in arteries often called "mild arterial stenosis" in biomechanical literature. This is the classic analytical approach based on the mild stenosis approximation, widely applied.This model treats blood flow as steady, laminar, and incompressible in a cylindrical tube, with a small axisymmetric narrowing stenosis, It uses a perturbation approach valid when the stenosis is mild.

7.Governing Equations of Steady Flow

Under mild stenosis approximation ($\delta/R_0 \ll 1$, slope of wall is small → lubrication-like or quasi-Poiseuille flow), the simplified momentum equation from Navier–Stokes in cylindrical coordinates, axial direction becomes:
$dp/dz = (1/r)\, d/dr\, [\, r\, \mu\, du/dr\,]$ (radial viscous term dominant)
With no-slip boundary condition: $u(r(z), z) = 0$ at $r = r(z)$
Integrating twice gives the velocity profile (parabolic-like, but radius varies slowly with z):
$u(r,z) = (1/(4\mu))\, (-dp/dz)\, [r(z)^2 - r^2]$
Volumetric flow rate Q is constant (continuity):
$Q = \int_0^{r(z)} 2\pi\, r\, u(r,z)\, dr = -\,(\pi\, r(z)^4 / (8\mu))\, dp/dz$
→ rearranged (Poiseuille form modified for varying radius):
$dp/dz = -\,(8\mu\, Q) / (\pi\, r(z)^4)$

8.Numerical Technique

The Finite Element Method (FEM) is used to solve various equation occurring during the modelling of mild stenoses in artery by dividing a domain into smaller elements. Its process typically unfolds in structured stages to approximate solutions to partial differential equations.processing discretization divides the continuous domain into finite elements like triangles or quadrilaterals, forming a mesh we define geometry, assign material properties, boundary conditions, and loads during this phase.Mesh refinement often targets high-stress areas for accuracy.SolutionElement stiffness matrices are derived and assembled into a global system of equations.Boundary conditions are applied, and the system is solved for primary unknowns like displacements using matrix inversion or iterative solvers. This core computation handles large-scale linear or nonlinear problems.Postprocessing Results such as stresses, strains, and pressure are computed from primary solutions and visualized via contours.

9.Results and Discussion

$\Delta = 0.0$: A flat line at y=1.0 represents a normal artery with no stenosis. $\Delta = 0.3$: A moderate dip reaching a minimum of 0.7 represents 30%narrowing.$\Delta = 0.6$: A deep dip reaching a minimum of 0.4 represents 60% narrowing.It was observed that in a mild stenosed artery defined as a reduction in arterial diameter of less than 50% represents a subtle but significant transition point in hemodynamics. While it often does not significantly restrict the total volume of blood flow at rest, it introduces localized disturbances that can accelerate the progression of cardiovascular disease.

• Velocity Profile

In a mild stenosed artery the cross-sectional area of the artery decreases, the velocity of the fluid must increase to maintain a constant flow rate.Even with a mild 20–30% narrowing, blood velocity increases at the the narrowest point of the stenosis.

• Pressure Changes

The increase in velocity at the stenosis throat comes at the expense of fluid pressure localised pressure drop as blood accelerates through the narrowing, static pressure drops.
•Pressure Recovery: Downstream of the narrowing, as the artery returns to its normal diameter, some pressure is recovered. However, in any stenosis, some energy is permanently lost to heat and friction.In mild stenosis, this pressure drop is usually negligible at rest but can become significant during exercise when flow rates increase.

• Wall shear stress

Wall Shear Stress is the frictional force exerted by blood flow on the endothelial lining of the artery. The increased velocity at the stenosis

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throat leads to high WSS. While extremely high stress can damage the vessel wall, the more critical changes often occurs

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