

# The Analysis of Reinforced Concrete Structure by Considering SSI in Various Soil at Different Location of Pile Foundation

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## Abstract

The interaction between soil and structure, known as Soil-Structure Interaction (SSI), plays a crucial role in the design and analysis of reinforced concrete (RC) structures, particularly in seismic-prone regions or areas with varying soil conditions. This study investigates the effects of SSI on an RC frame building with pile foundations, considering three different soil types: hard, medium, and soft. To simulate the behavior of the soil in the static loading condition the Mohr-Coulomb model was used and finite element program ANSYS was used for the analysis. The results of the vertical reactions, the horizontal reactions and the bending moments at the foundation level, that is at corner, edge and central pile locations, are investigated with respect to the SSI occurrences.

The results show that the soil type has highly significant effect on the structural response. The vertical reactions at corner, and edge piles in soft soil increased due to increased settlement and load redistribution; central piles had lower vertical reaction. As a result of the low stiffness and high compressibility of the soft soil that amplifies lateral forces and moments and reduces vertical reactions, horizontal reactions and bending moments were highest for soft soil and particularly for edge piles. On the contrary, the fixed base model without SSI

under predicted these reactions and moments, implying the shortcomings of conventional design that omit soil flexibility.

The study shows that SSI should be included in the RC structure design, particularly in soft soil conditions, to give accurate prediction of structural behavior and avoid risk of excessive settlement, lateral deformation and foundation rotation. These findings are practical and impart pile foundation design with practical consideration and highlight the need of specialized design approach depending on the soil conditions in order to improve structures' safety and stability. Both normative and empirical contributions to the SSI literature are made by this research, and recommendations for improving the performance of RC structures in different soils are given to the engineer.

**Keywords:** Soil-Structure Interaction, RC Frame, Hard Soil, Medium Soil, Soft Soil, Different Location of Pile Foundation.

## 1. Introduction

### 1.1 Background

Applications of reinforced concrete (RC) structures include strong, durable and versatile structures [1] because of which RC structures are widely used in civil engineering. The interaction between the foundation and the under soil is a

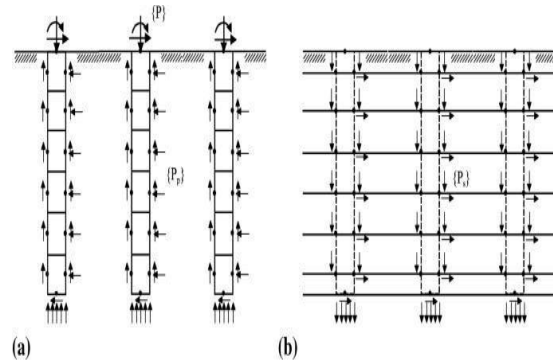
phenomenon very much depending on these structures, called Soil Structure Interaction (SSI) [2]. The dynamic response, stability and behavior of RC structures in seismic-prone regions or under heavy loading conditions are very much dependent on SSI [3]. Special attention is given to the foundation system, especially pile foundations, being the main component that transfers the structural loads to the ground. Designing them has to take into account soil variability, e.g., stiffness, density, and damping properties that change from one site to another [4].

In most cases, traditional structural analysis is based on a fixed-base condition, without regarding the SSI effects. However, this assumption yields inaccurate predictions of structural behavior due to the flexibility of the soil causing very large changes in natural frequencies, mode shapes, and stress distribution in the structure [5]. More recently, associated studies have indicated that SSI must be part of the RC structure designing and analysis process in the designs of tall buildings, bridges, and offshore structures, where the interaction between the soil and foundation is more noticeable [6].

In particular, pile foundations are commonly used in soft or adverse soils, and this has a higher potential for the dependence of pile foundation behavior on the soil type and the location of the pile in the soil profile [7]. The stiffness and damping of different soil types, including clay, sand, and silt, vary depending on the degree and thus can impact the load-bearing capacity and settlement behavior of the pile [8]. Additionally, the pile's position, whether near the surface or buried deeper into the soil, would also influence its interaction with the surrounding soil and the overall structural response [9].

However, the importance of SSI has not been recognized to the extent that there have been no comprehensive studies that would systematically study the effects of SSI on RC structures with different soil types and pile locations [10]. To date, most of the work has been restricted to special soil

conditions or simple foundation models, which do not capture reality [11]. This knowledge gap implies the need for further detailed investigation into how the behavior of RC structures is affected by varying soil properties and pile configurations [12].



**Figure-1: Soil Structure Interaction with Pile Foundation with different Load.**

#### 1.1.1. Pile Foundation

Deep foundation systems intended to transfer structural loads from weak or compressible soil layers to stronger, more stable strata via smaller loads through deeper soil layers are known as pile foundations [13]. Yet, in situations where shallow foundations are not feasible, for reasons of poor soils, high groundwater levels, or large structural loads, they are essential [14]. Materials used to make piles include reinforced concrete, steel, or timber, and they are installed using driving, drilling, or jacking into the ground [15].

Piles may be categorized based on their load-transferring mechanism: end-bearing piles are founded on a firm soil or rock layer at the pile toe, and friction piles are founded on the resistance along the pile skin [16]. Pile type has to be selected based on soil properties, load requirements, and site conditions [17]. Pile foundations are used extensively in high-rise buildings, bridges, offshore structures, and infrastructure construction in seismic zones [18].

Accelerations building up in soil structure, soil stiffness, soil density, and damping play an important role in the dynamic behavior of

pile foundations, and their design must take them into account [19]. Load distribution and settlement are also affected by pile length, diameter, spacing, and arrangement [20]. Optimization of pile design in the presence of both static and dynamic loading conditions is done with the help of advanced analytical and numerical methods to guarantee stability and reduced deformation [21].

### 1.1.2 Different Types of Soil

In geotechnical engineering, soil classification with respect to stiffness and strength is fundamental because a considerable decision whether or not a foundation or structure can be safely designed and performed depends on their nature [22]. There are three kinds of soils, namely, soft, medium, and hard soils, each with different mechanical properties and behavior under loading [23].

**Soft Soil:** The shear strength is low, compressibility is high, and stiffness is low in soft soils like loose sand, silty clay, or organic deposits [24]. These soils have low bearing capacity and large settlements under load [25]. In addition, structures built on them are more vulnerable in case of earthquakes due to the high amplification of seismic waves [26]. For example, it is common that large soft soils which have unsaturated challenges such as liquefaction are associated with instability and require ground improvement or deep foundations for stability [27].

Medium soils have moderate shear strength and stiffness, such as well graded gravel, stiff clay and dense sand [28]. In between load bearing capacity and flexibility, on the other hand, they are frequently used for a variety of construction projects [29]. The seismic forces are only moderately amplified and cumulative settlement is less in these soils compared with soft soils [30].

The issues related with medium soils can be alleviated by proper foundation design [31].

**Hard soils:** They have high shear strength, low compressibility, high stiffness [32]. Dense gravel, cemented soils, or weathered

rock are the formation of these soils [33]. Such structures are good in terms of load carrying capacity and small settlement [34]. In case of hard soil, low seismic wave amplification reduces the risk of structural damage during earthquakes [35]. The foundations on the hard soils will be more stable and cost effective [36].

### 1.2. Mohr-Coulomb Soil Model

The Mohr Coulomb soil model [37] is used widely to describe the shear strength and failure behavior of soils. Specifically, there is a particular relevance to this when analyzing RC buildings on different types of soil. The model provides a useful, simplified, though effective, model for understanding soil response to stress and deformation and is necessary to evaluate soil-structure interaction [38].

The Mohr-Coulomb model is based on two key parameters: cohesion ( $c$ ) and the angle of internal friction ( $\phi$ ). These parameters define the shear strength of the soil in terms of a linear failure criterion:

$$\tau = c + \sigma_n \tan(\phi)$$

It is supposed that soil failure occurs when shear stress on a plane reaches shear strength as given above. Both the effect of pore water pressure, which can reduce the effective stress and hence the shear strength of the soil and the effect of grouting agent itself is also considered [39].

For the SSI analysis of RC buildings, the Mohr-Coulomb model is applied to simulate the soil behavior occurring under static and dynamic loading cases for the soft, medium, and hard soils. For example: Deformation and low shear strength characterized soft soils (low  $c$  and  $\phi$ ) and thus high deformation and low shear strength contribute to significant settlement and large increase of seismic forces [40].

Medium soils (moderate  $c$  and  $\phi$ ), balance deformation and strength and are used in most construction projects.

The high  $c$  and  $\phi$  coefficients displaying hard soils with little deformation and high

shear strength provide good stability for the structure.

### 1.3. Problem Statement

Lots of the scholar have studied the effect of the pile foundation, because the type of RC structure such as circular, square or rectangular, and the negative skin friction (NSF) due to vertical loading, and the central pile foundation of the structure. Now, we will see the effect of horizontal force, vertical force, and bending moment of the footing using Mohr-Coulomb Soil Model with Ansys workbench.

### 1.4. Objectives of the Study

The primary objectives of this research paper are as follows:

**To Analyze Soil-Structure Interaction (SSI):** The study aims to investigate the soil-structure interaction of a G+3 reinforced concrete (RC) building founded on different types of soils (hard, medium, and soft) using the Mohr-Coulomb soil model. The focus is on understanding how the interaction between the soil and the structure affects the overall behavior of the building under gravity loads.

**To Compare Flexible Base and Fixed Base Models:** The research seeks to compare the responses of a flexible base model (considering soil-structure interaction) with a fixed base model (ignoring soil-structure interaction) to evaluate the impact of SSI on the building's structural performance.

**To Evaluate Foundation Reactions:** The study aims to analyze the effect of SSI on various foundation reactions, including vertical reactions, horizontal reactions, and bending moments at the footing level, for different types of soils.

**To Validate the Finite Element Model:** The study aims to validate the finite element model used in ANSYS software by comparing the results with established theoretical concepts, such as the immediate settlement calculation from IS 8009-1-1976.

**To Provide Practical Insights for Design:** The research intends to provide practical insights and recommendations for the design of RC buildings, emphasizing the importance of considering soil-structure interaction, especially in soft soil conditions, to ensure structural safety and performance.

## 2. Methodology of Research

### 2.1. System Description

In this research work, we have used Ansys workbench for modeling the G+3 RC frame structure with pile foundation by using different load such as dead, live, floor, finishing, and wall load. These details are given below:

#### 2.2.1. Material Used

In this research work, we have used different grade of the materials for steel bar, concrete in the RC Frame Structure as well as in the Pile Foundation, The details of grade of the material is given below:

**Table-1: Materials**

Serial Number	Grade of the Materials	Used in the modes.
1.00	Concrete Grade for Beam, and Slab.	M25 Grade.
2.00	Grade of Steel bar for Main Reinforcement.	Fe500
3.00	Grade of Steel bar for Transverse Reinforcement.	Fe415
4.00	Concrete Grade for Column, and Pile Foundation.	M30 Grade.

### 2.2.2. Geometry of the Model

In this research work, we have used Beam,

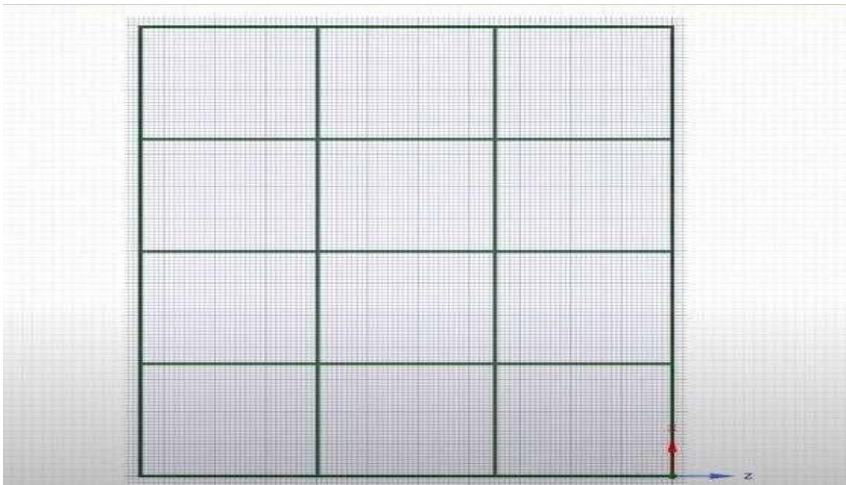
Column, Slab, plan area of the structure, height of the structure. These types of the **Table-2: Building Geometry.**

details are given below in the form of the table:

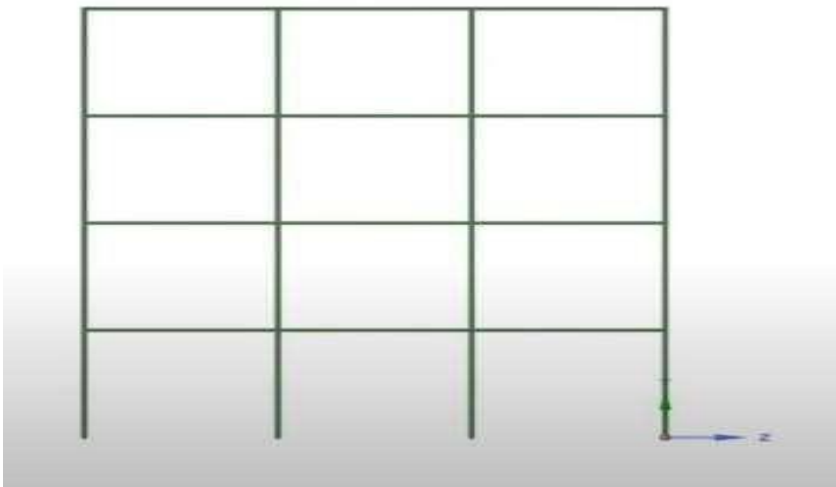
Serial Number	Geometry of Model	Dimensions.
1.00	Width of Model in X-Direction	3250 mm
2.00	Width of Model in Y-Direction	3000 mm
3.00	Height of Each Floor	3000 mm
4.00	Size of Beam	300 mmX400 mm
5.00	Size of Column	400 mmX400 mm
6.00	Thickness of Slab	140 mm

The details view of the model such as top view, elevation, and 3D view model, and

details view of the pile foundation are given below:



**Figure-2: Plan View of Model**

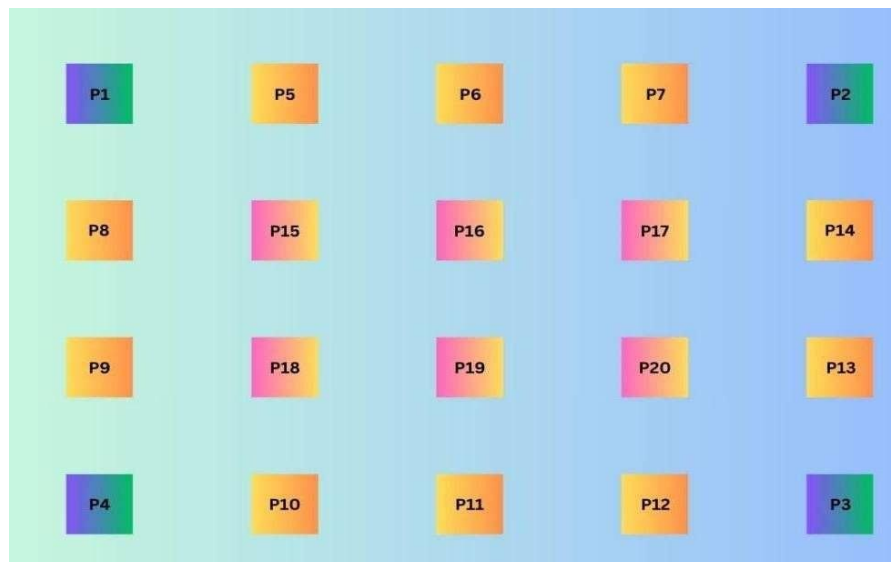


**Figure-3: Elevation of Model.**





**Figure-4: 3D View of Models.**



**Figure-5: Pile Foundation Location.**

As we can see in the figure-5, the P1, P2, P3, and P4 are corner pile foundation, P5 to P14 is edge pile foundation, and P15 to P20 is central pile foundation.

### 2.2.3. Loading Conditions on G+3 Structure.

The present study focuses on analyzing a G+3 RC frame structure considering only dead and live loads. The dead load is based on the self-weight of the structural members,

calculated using the density of the materials and following the guidelines of IS 875 Part 1 (1987). The live load is determined according to IS 875 Part 2 (1987). These loads are then applied to the structural members using ANSYS to simulate real-world conditions. The soil properties used in the analysis are sourced from various literature based on their relevance to soil-structure interaction. Soil property selection is performed with respect to the footing

model in ANSYS, examining the soil's response to changes in individual parameters while keeping others constant. The soil properties chosen for this study provided consistent, standard-patterned responses.

Three different soil types are considered to study soil-structure interaction, and behavioral observations are made to analyze the problem at hand.

**Table-3: Loading Condition at G+3 RC Frame Structure.**

Serial Number	Type of Loading	Value/ IS Code
1.00	Imposed Load	3KN/m <sup>2</sup> / IS 875 part:2
2.00	Dead Load	IS 875 Part:1
3.00	Floor Finishing Load	1KN/m <sup>2</sup> / IS 875 part:1

**Table-4: Property of Soil.**

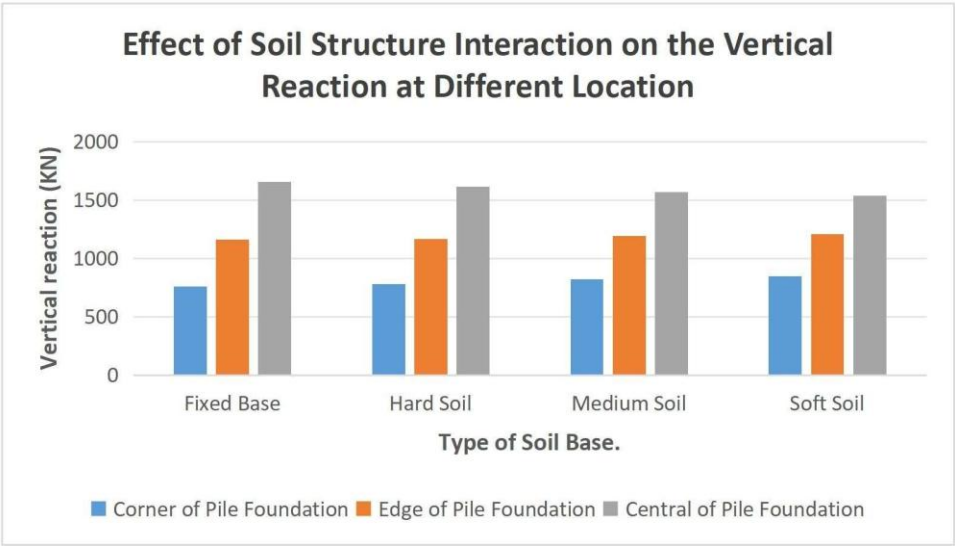
Serial Number	Property of Soil	Hard Type Soil	Medium Type Soil	Soil Type Soil	Unit
(a)	Soil Density.	2050	1790	1646	Kg/m <sup>3</sup>
(b)	Modulus of Elasticity.	75.00	33.00	22.00	MPa
(c)	Poisson's ratio.	0.35	0.30	0.25	
(d)	Shear modulus.	27.78	12.69	8.80	MPa
(e)	Bulk modulus.	83.33	27.5	14.67	MPa
(f)	Cohesion.	0.1	0.1	0.1	MPa

### 3. Analysis of Result

#### 3.1. Vertical, Horizontal, and Bending Moment by effect of Soil Structure Interaction at different type of the foundation.

In this section of the analysis of the results, we have analysed the horizontal reaction of the pile foundation, vertical reaction of the pile foundation, and bending moment of the pile foundation of the pile foundation by effect of the soil structure interaction at the different type of the soil such as soft soil,

medium soil, and hard soil. Here the graph of the Vertical reaction of the pile foundation by the effect of the soil structure interaction at the different type of the soil.



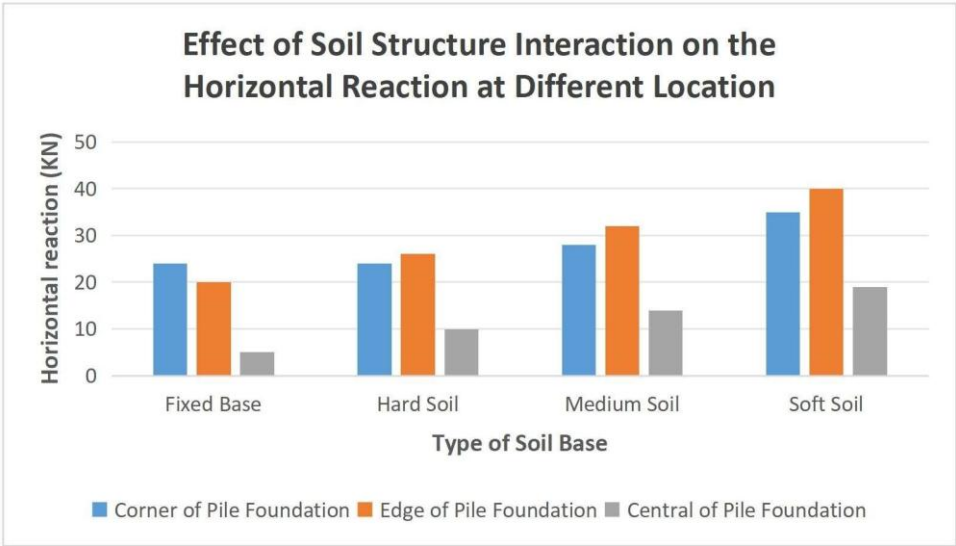
**Graph-1: Vertical Reaction on different location of pile foundation due to effect of SSL.**

As we can see from the graph-1, the maximum vertical reaction is in the fixed base at the central location of the pile foundation, and minimum vertical reaction at the fixed base of the corner location of pile foundation.

Now, the horizontal reaction of the pile foundation at the different location in different type of soil base are given below in the form of the table as well as graph:

**Different location of pile foundation due to effect of SSL.**

As we can see from the table and graph, the maximum horizontal reaction at the edge of the pile foundation in the soft soil, and minimum horizontal reaction is in the fixed base of the central location of the pile foundation.



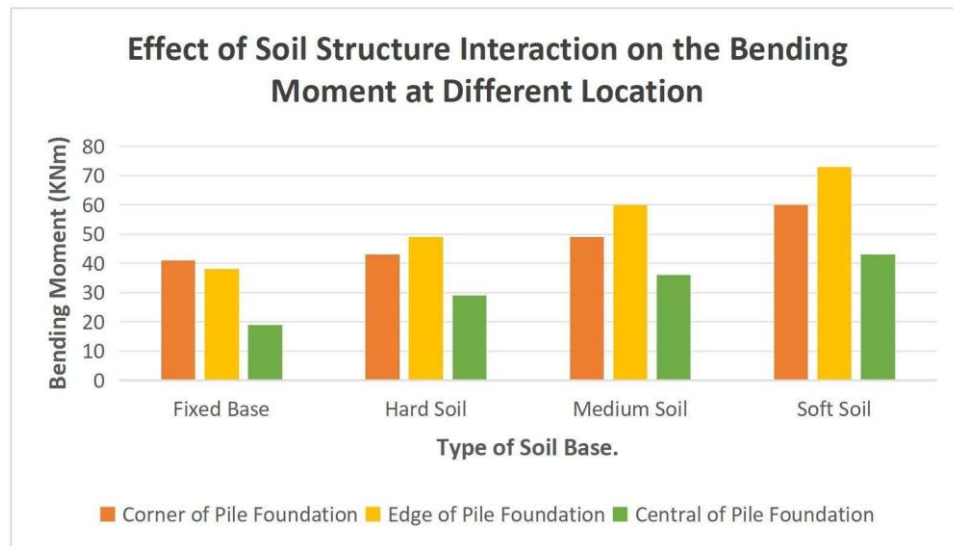
**Graph-2: Horizontal Reaction on**



Lastly, the bending moment of the pile foundation at the different location in different type of soil base are given below in the form of the table as well as graph:

#### 4.1. Vertical Reactions

In the fixed base model, the maximum vertical reactions at the pile foundations were found at the central location of 1655 kN. This is expected, because the most



**Graph-3: Bending Moment on different location of pile foundation due to effect of SSI.**

From the table and graph above, we can see that the maximum of bending moment is the edge of pile foundation in soft soil, the minimum of bending moment is in the central of pile foundation on the fixed base of soil.

#### 4. Conclusion

The study was to investigate the affect of Soil Structure interaction on a G+3 reinforced concrete structure with pile foundations for three different soils (hard, medium and soft) by using Mohr Coulomb soil model. It has been shown that the vertical reactions, horizontal reactions, and bending moments at the foundation level are greatly different in the soil and the perimeter, corner, edge and centrally located pile foundations cases. The results from these findings indicate that SSI is an important factor that should be taken into consideration during the design and analysis of RC structures, especially in regions prone to earthquakes or those that have different soil conditions.

loaded piles are located in the load distribution system and for this reason often bear the most load in general. However, when SSI was considered, the lower stiffness of the soil resulted in a decrease of the vertical reactions. For example, the central pile reacted vertically, 1536 kN in soft soil, less than 1614 kN in hard soil and 1568 kN medium soil. The source of the reduction is due to the soft soil being more flexible and settle more easily, redistributing the load over the foundation more evenly as opposed to concentrating the stress at the central piles.

As the soil stiffness decreased, it increased the vertical reactions at the corner and edge piles. For instance, in hard soil the corner pile reaction increased from 780 kN to 847 kN in soft soil. Thus, this behavior can be explained by the fact that softer soils allow greater settlement and redistribution of loads, whereby the corner and edge piles tend to endure more load, as the central piles tend to settle more significantly.

#### 4.2. Horizontal Reactions

Overall size, contours, and the horizontal reactions at the edge were significantly

higher (40 kN) in soft soil and also in general where they were very high. Higher factor of safety develops due to lower shear strength and stiffness of soft soil, which permit greater lateral movement when they are loaded. On the contrary, as soil flexibility is not accounted for via the fixed base assumption, the level of horizontal reactions in the fixed base model (5 kN at the central pile) were minimal (5 kN at the central pile).

SSI is important in seismic, dynamic loading and metropolitan areas as the increase in horizontal reactions in softer soils demonstrates the need to consider soil SSI in these conditions. It is also evident that if the horizontal reactions are high, the structures on such soils are more likely to be affected by lateral deformations and structural damage can occur if the effects of lateral deformations are not adequately considered in the design.

#### 4.3. Bending Moments

Similar trend of the bending moments was the horizontal reactions of the edge piles in soft soil (73 kNm). The reason is that the edge piles are located at the braiding of the structure, and more exposed to lateral forces and moments. As in the fixed base model, the bending moments were the lowest (19 kNm at the central pile), which confirms the unsuitability of the fixed base assumption in modeling the structural behavior under SSI. Our critical finding is that the increase in bending moment in soft soils is a strong indication that structures on soft soils are more susceptible to foundation rotation, tilting, resulting into un-even settlement of the iii structural instability. This justifies even more the need to think carefully about the SSI when designing foundations in the soft soil condition.

#### 4.4. Practical Implications

Implications for the design of reinforced concrete (RC) structures based on the results of this study are quite significant. The first focus of the study is to point out the criticality of soil—structure interaction (SSI)

in design, which, by neglecting, leads to significant inaccuracy in predicting the structural behavior, particularly in soft soil conditions. Therefore, engineers need to take a measure of SSI to verify safety and stability of structures in seismic areas. Secondly, behavior of structures with respect to soil suffers from significant variation, especially in the case of soft soil which possesses lesser stiffness and larger compressibility causing settlements, lateral forces and bending moments to increase. In such cases, ground improvement techniques or deeper foundation may be needed. The study also shows the importance of designing the pile foundation properly considering soil conditions and location of the pile, as it points out that edge and corner piles may need extra reinforcement or even larger diameters in order to be able to withstand higher lateral forces and bending moments for soft soils. In addition, the finite element model developed in this research has been validated by comparing to the immediate settlement equations of IS 8009-1-1976 for parallel validation of the model. This research concludes that SSI should be taken into account in the design and analysis of RC structures and provides useful information in regard to the effects of different soil types and pile locations on the performance of the structure. When SSI is synthesised into the design process, engineers will be able to create safer, more stable, and more functional structures under difficult soil conditions.

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