Stochastic Analysis of Two Non-Identical units System Model Subject to Inspection Policy

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Abstract
The purpose of the present paper is to analyse a two non-identical units system model in which unit A is in operative mode and unit B is in standby mode. Two repairmen expert and ordinary are considered to repair the failed units. Unit A is repaired by expert repairman who is not always available with the system while unit B by ordinary repairman which is always with the system. Expert repairman takes some significant time to repair the failed unit and after the repair, inspection is carried out to ascertain whether the repair is perfect or not. If repair is perfect then the repaired units becomes operative otherwise the repair is referred to the expert repairman. The failures of the unit are independent and the failure time distribution of both the units are taken as general. All random variables are statistically independent. Using regenerative point technique we find the following characteristics of the system such as Transition Probabilities and Mean Sojourn times, Mean time to system failure (MTSF), Availability of the system, Busy Period for Expert and ordinary repairman, Expected number of Repairs of the unit, Expected number of Replacement of the unit, Expected number of Inspection of the unit, Net Expected Profit earned by the system during the interval (0,t) and in steady state.

Keywords: Repair, Replacement, Inspection

1.1.Introduction

The technique of redundancy has been used extensively in many industrial systems in order to improve their performance in terms of reliability and mean life. Repair maintenance is one of the important measures for increasing the effectiveness of a system. Many authors including [1,2,4,5,9]have analysed two unit system models considering different repair policies viz. two types of repair, inspection, post repair, after the repair, preparation time for repair, expert and regular repairman with patience time of regular repairman etc. Therefore, several research papers have been investigated and analyzed by various authors in this direction under different repair and operation policies. Vibha Goyal and K. Murari 6 analysed two unit standby system with two types of repairman. Keeping in view the fact that the expert repairman takes some time to repair the failed unit, Gupta .R, C.K Goel and A.Tower 2 developed a system model with administrative delay in repair and correlated lifetimes.Further, Bhatti .J, Ashok Chitkara and Nitin Bhardwaj 4 have analyzed a system with inspection and two types of failure. Also, Bashir .R, JP Singh Joorel and Ranjeet Kour[3] developed a two unit system model with repair and inspection policies. Chander. S [5] worked on different types of inspection subject to inspection for on line repair and replacement.

The purpose of the present paper is to analyze a two non identical unit system model consisting of two units A and B. Unit A is in operative mode and unit B is in standby mode. Two repairmen expert and ordinary are considered to repair the failed units. Unit A is repaired by expert repairman who is not always available with the system while unit B

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is repaired by ordinary repairman which is always available with the system. Expert repairman takes some significant time to repair the failed unit and after the repair, inspection is carried out to ascertain whether the repair is perfect or not. If repair is perfect then the repaired units become operative otherwise the repair is referred to the expert repairman. The failures of the units are independent and the failure time and repair time distribution of both the units are taken as exponential. All random variables are statistically independent.
Using semi- Markov process and regenerative point technique, the following measures are obtained-

1. Transition Probabilities and Mean Sojourn times.
2. Reliability and Mean time to system failure (MTSF).
3. Expected uptime and downtime of the system.
4. Busy period for expert repairman and ordinary repairman.
5. Expected number of repairs by expert and ordinary repairman.
6. Expected number of replacement of the unit.
7. Net expected profit earned by the system during the interval (0,t) and in steady state.

1.2. Model Description and Assumptions

1. The system comprises of two units- A and B. Initially unit-A is in operative mode and unit-B is in standby mode.
2. Two repairmen are available to repair the failed unit i.e. Expert and ordinary repairman. The expert repairman repairs the unit-A who takes some significant time (a random variable) to repair the failed unit while ordinary repairman is always available with the system which repairs the unit-B.
3. Upon the failure of unit A, repairman takes some time (delay time) to reach the system. After repair, inspection is carried out to ascertain whether the repair is perfect or not. If repair is perfect it goes back to the operating mode otherwise it is replaced by the expert repairman.
4. However, if unit B fails it is repaired by ordinary repairman who is always available in the system and after repair the unit becomes operative.
5. During the repair of the unit A by the expert, if the unit B also fails then the repair of the unit-B is also done by the expert, so the unit B has to wait for the repair until the repair of the unit A is not completed.
6. The failures and repairs of the units are independent and the failure time distributions of the units are taken as Exponential.
7. The repair time distribution of the units are taken as exponential.

1.3. Notations and States of The System

We define the following symbols for generating the various states of the system.

A_0/B_0: Unit A /unit B is in operative mode.

B_s: Unit B is in standby mode.

A_d: Administrative delay time of unit A.

A_{re} /B_r : Unit A/unit B under repair.

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B_{wr}: Unit B is waiting for repair.

A_I/A_R: Unit A under inspection/ replacement of the unit.

Thus considering the above symbols in view of the assumptions stated, the possible states of the system are as follows**Up states:**

$S_0 = (A_0, B_S)$ $S_1 = (A_d, B_0)$ $S_3 = (A_{re}, B_0)$

$S_4 = (A_I, B_0)$ $S_6 = (A_R, B_0)$ $S_9 = (A_0, B_r)$

Failed states:

$S_2 = (A_d, B_r)$ $S_5 = (A_{re}, B_{wr})$ $S_7 = (A_I, B_{wr})$

$S_8 = (A_R, B_{wr})$

b)	**NOTATIONS:**
E	**: Set of regenerative states** **= {$S_0, S_1, S_3, S_4, S_6, S_9$}**
E	**: Set of non – regenerative states** **= {S_2, S_5, S_8, S_7}**
α_1	**: Failure rate of unit A.**
α_2	**: Inspection Rate of unit A.**
α_3	**: Repair Rate of unit A.**
γ_1	**: Completion rate of Inspection of unit A .**
p/q	**: Probability with which unit will get repaired/replaced.**
γ_2	**: Replacement Rate of unit A.**
β_1	**: Failure Rate of unit B.**
β_2	**: Repair Rate of unit B.**

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Transition Diagram

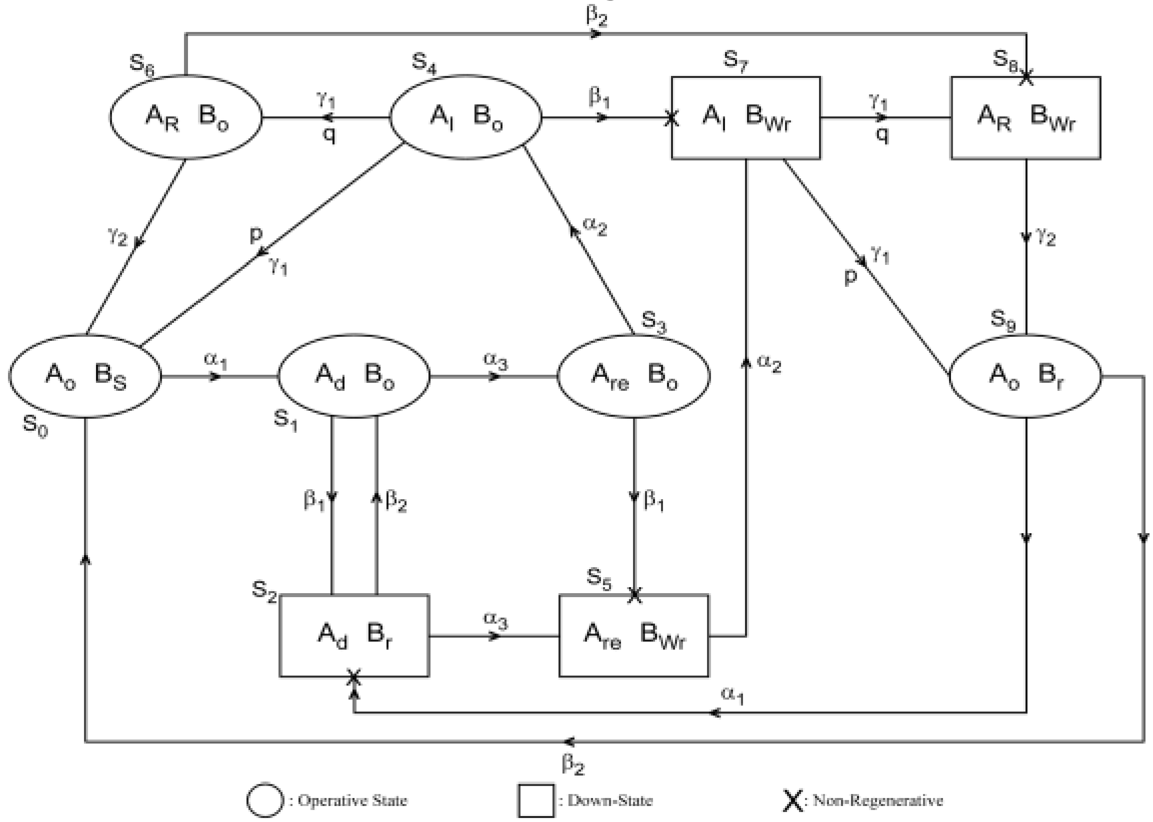

Fig 1.1

1.4.Transition Probablities

Let denotes the state visited at epoch + just after the transition at , where 1, 2.... represents the regenerative epochs, then{, } constitute a Markov-Renewal process with state space E and

$= ( )_{+1} = [\; _{+1} - \; \leq | =]$is the semi Markov

kernel over E.

Then the transition probability matrix of the embedded Markov chain is

$P = p_{ij} = Q_{ij}(\infty) = Q(\infty)$

The steady state transition probabilities are given by:

$p_{01} = \alpha_1 \; e^{-\alpha_1 u} du = \frac{\alpha_1}{\alpha_1} = 1$ $\quad p_{12} = \beta_1 \; e^{-(\beta_1 + \alpha_3)u} du = \beta_1$

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α_1 $\overline{(\beta_1+\alpha_3)}$

p_{13} $\overline{(\beta_1+\alpha_3)}$ $\overline{(\beta_2+\alpha_3)}$ $=\gamma_1 p \int e^{-(\gamma_1+\beta_1)}$ $= p_{21} = p_{25} =$ α_3 β_2 α_3 $\overline{(\beta_2+\alpha_3)}$

$\frac{\alpha_2}{(\alpha_2+\beta_1)}$ $\frac{\gamma_1 q}{(\gamma_1+\beta_1)}$

p_{34} $\frac{\gamma_2}{(\gamma_2+\beta_2)}$ $= p_{40} u\, du =)\gamma_1\gamma+1p\beta_1$ p_{46}

$= p_{57} = 1$ $p_{60} = p_{78} = p$ p_{79} $\frac{\beta_2}{(\alpha_1+\beta_2)}$ $= q$ $p_{89} = 1$ $p_{90} =$

The indirect transition probability may beobtained as follows:

$p_{(5)37} = 1 - \alpha_2\alpha+2\beta_1 = \alpha_2\beta+1\beta_1$ ( ) ( )

$p_{(7)48} = q - \frac{\gamma q}{(\gamma_1+\beta_1)} = \frac{\beta q}{(\gamma_1+\beta_1)}$ 1 1 $\frac{\alpha_1\beta_2}{(\beta_2+\alpha_3)(\beta_2+\alpha_1)}$

$p_{497} = p - \frac{\gamma p}{(\gamma_1+\beta_1)} = \frac{\beta p}{(\gamma_1+\beta_1)}$ 1 1 $\frac{\alpha_1\alpha_3}{(\beta_2+\alpha_3)(\beta_2+\alpha_1)}$

$p_{(8)69} = 1 - \overline{(\gamma_2\gamma+2\beta_2} = \overline{(\gamma_2\beta+2\beta_2}$ $p_{(2)91} = p_{(2)95} =$

It can be easily verified that	
$p_{12} + p_{13} = 1$ $p_{21} + p_{25} = 1$	$p_{34} + p_{(5)37} = 1$
$p_{40} + p_{46} + p_{(7)48} + p_{(7)49} = 1$	$p_{01} = p_{57} = p_{89} = 1$

( )

$p_{60} + p_{68} = 1$ $p_{78} + p_{79} = 1$ $p_{90} + p_{912} + p_{(2)95} =$

1 A) Mean sojourn times:

The mean sojourn time in state S_i denoted by μ_i is defined as the expected time taken by the system in state S_i before transiting to any other state. To obtain mean sojourn timeμ_i, in state S_i, we observe that as long as the system is in state S_i, there is no transition from S_i to any other state. If T_i denotes the sojourn time in state S_i then mean sojourn time μ_i in state S_i is:

$\mu_i = E[T_i] = \quad P(T_i >)dt$

Therefore,

$\mu_0 = e^{-\alpha_1 t}dt = \alpha\_1 1$ $\mu_1 =$ $\overline{(\alpha_3+\beta_1)}$ 1 $\mu_2 =$ 1

$\overline{(\beta_1+\ _2)}$ $\frac{1}{(\ _2+\beta_2)}$ $\frac{1}{(\beta_2+\ _1)}$ $\mu_3 =$ 1 $\frac{1}{(\ _1+\beta_1)}$ $\underline{1}$ 1 $\overline{(\alpha_3+\beta_2)}$ $\underline{1}$ $\mu_4 = \mu_5 = \alpha_2$

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$\mu_6 = \mu_7 = \gamma_1$ $\mu_8 = \gamma_2$

$\mu_9 =$

1.5 Analysis of Reliability and MTSF

Let the random variable T_i denotes the time to system failure when system starts up from state $S_i \in E$. Then the reliability of the system is given by

$R_i(t) = P[T_i > t]$

Taking their Laplace Transform and solving the resultant set of equations for $R^*_0(s)$, we get

$$R_0^*(s) = \frac{N_1(s)}{D_1(s)} \qquad \textbf{(1.5.1)}$$

Where

$$N_1(s) = Z^*_0 + q^*_{01}[Z^*_1 + q^*_{13}(Z^*_3 + q^*_{34}(Z^*_4 + q^*_{46}Z^*_6))]$$

$$D_1(s) = (1 - q_{01}^* q_{13}^* q_{34}^* (q_{40}^* + q_{46}^* q_{60}^*))$$

Taking the Inverse Laplace Transform of **(1.5.1)**, one can get the reliability of the system when it starts from state $_0$.

To get MTSF, we use the well known formula

$$E(T_0) = \int R_0(t)\,dt = \lim_{s\to 0} R_0^*(s) = N_1(0)/D_1(0)$$

where,

$$N_1(s) = \mu_0 + p_{01}[\mu_1 + p_{13}(\mu_3 + p_{34}(\mu_4 + p_{46}\mu_6))]$$

$$D_1(s) = 1 - p_{01}p_{13}p_{34}(p_{40} + p_{46}p_{60})$$

Since, we have $q^*_{ij}(0) = p_{ij}$ and $\lim_{s\to 0} Z_i^*(s) = \int Z_i(t)\,dt = \mu_i$

1.6. Availability Analysis

Let $A_i(t)$ be the probability that the system is in operative mode at epoch t, when it initially starts from $S_i \in E$. To obtain recurrence relations among different pointwise availabilities we use the simple probabilistic arguments.

Solving the resultant set of equations and simplifying for $A^*_0(s)$, we have

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$A_0^*(s) = N_2(s)/D_2(s)$ **(1.6.1)**

Where

$N_2(s) = {}_{*0}\left[\left(1 - q_{95}^{(2)*} q_{57}^* (q_{78}^* q_{89}^* + q_{79}^*)\right)(1 - q_{12}^* q_{21}^*) - \left\{ q_{13}^* q_{34}^* \left(q_{46}^* q_{69}^{(8)*} + q_{48}^{(7)*} q_{89}^* + q_{49}^{(7)*} \right) q_{91}^{(2)*} + q_{13}^* q_{37}^{(5)*} (q_{78}^* q_{89}^* + q_{79}^*) q_{91}^{(2)*} + q_{12}^* q_{25}^* q_{57}^* (q_{78}^* q_{89}^* + q_{79}^*) q_{91}^{(2)*} \right\}\right] + q_{01}^* \left[1 - q_{95}^{(2)*} q_{57}^* (q_{78}^* q_{89}^* + q_{79}^*)\right]$

$q_1^* + q_{13}^* \,{}_3^* + q_{34}^* \,{}_4^* + q_{34}^* q_{46}^* \,{}_6^* + q_{01}^* \left(q_{13}^* q_{34}^* \left(q_{46}^* q_{69}^{(8)*} + q_{48}^{(7)*} q_{89}^* + q_{49}^{(7)*} \right) q_{91}^{(2)*} + q_{13}^* q_{37}^{(5)*} q_{78}^* q_{89}^* + q_{79}^* q_{91}^{(2)*} + q_{12}^* q_{25}^* q_{57}^* q_{78}^* q_{89}^* + q_{79}^* q_{91}^{(2)*} \right) {}_9^*$ **(1.6.2)**

and

$D_2(s) = \left(1 - q_{95}^{(2)*} q_{57}^* (q_{78}^* q_{89}^* + q_{79}^*)\right)(1 - q_{12}^* q_{21}^*) - \left\{ q_{13}^* q_{34}^* \left(q_{46}^* q_{69}^{(8)*} + q_{48}^{(7)*} q_{89}^* + q_{49}^{(7)*} \right) + q_{13}^* q_{37}^{(5)*} (q_{78}^* q_{89}^* + q_{79}^*) + q_{12}^* q_{25}^* q_{57}^* (q_{78}^* q_{89}^* + q_{79}^*) \right\} q_{91}^{(2)*} - q_{01}^* \left(1 - q_{95}^{(2)*} q_{57}^* (q_{78}^* q_{89}^* + q_{79}^*)\right)\left[q_{13}^* (q_{34}^* q_{40}^* + q_{34}^* q_{46}^* q_{60}^*) \right] + q_{01}^* \left[q_{13}^* q_{34}^* \left(q_{46}^* q_{69}^{(8)*} + q_{48}^{(7)*} q_{89}^* + q_{49}^{(7)*} \right) + q_{13}^* q_{37}^{(5)*} (q_{78}^* q_{89}^* + q_{79}^*) + q_{12}^* q_{25}^* q_{57}^* (q_{78}^* q_{89}^* + q_{79}^*) \right] q_{90}^*$ **(1.6.3)**

The steady state availability is given by

$A_0 = \lim_{t\to\infty} A_0(t) = \lim_{s\to 0} sA_0^*(s) = \frac{N_2(0)}{D_2'(0)}$

As we know that, $q_{ij}(t)$ is the pdf of the time of transition from state S_i to S_j and $q_{ij}(t)\,dt$ is the probability of transition from state S_i to S_j during the interval $(t, t + dt)$, thus

$\lim_{s\to 0} {}^*_i(s) = \int {}_i(t)\,dt = \mu_i$ and $q^*_{ij}(s) = q_{ij}^*(0) = p_{ij}$, we get

Therefore,

$N_2(0) = \mu_0 \left[\left(1 - p_{95}^{(2)} p_{57} (p_{78} p_{89} + p_{79})\right)(1 - p_{12} p_{21}) - \left\{ p_{12} p_{25} p_{57} (p_{78} p_{89} + p_{79}) p_{91}^{(2)} + p_{13} p_{34} \left(p_{46} p_{69}^{(8)} + p_{48}^{(7)} p_{89} + p_{49}^{(7)} \right) p_{91}^{(2)} + p_{13} p_{37}^{(5)} (p_{78} p_{89} + p_{79}) p_{91}^{(2)} \right\}\right] + p_{01}\left(1 - p_{95}^{(2)} p_{57} (p_{78} p_{89} + p_{79})\right)\left(\mu_1 + p_{13}\mu_3 + p_{34}\mu_4 + p_{34} p_{46}\mu_6\right) + p_{01}\left(p_{12} p_{25} p_{57} (p_{78} p_{89} + p_{79}) + p_{13} p_{34} \left(p_{46} p_{69}^{(8)} + p_{48}^{(7)} p_{89} + p_{49}^{(7)} \right) + p_{13} p_{37}^{(5)} (p_{78} p_{89} + p_{79}) \right)\mu_9$ **(1.6.4)**

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and

D2 0 = 1 − p952 p57 p78 1 − p12p21) − [p12p25p57 (p78p89 + p79 p912 + p89 + p79) ( ) ( ) +

p13p34 p46p698 + p487 + $p_{49}^{(7)}$) $p_{91}^{(2)}$ p89+ p13p375 p78p89 + p79] p912 (− p01 1 − (p952

p57 p78p89 + p79)] − p01 [p12 ( ) (p13p34 p40 + p46p60p25p57 p78p89 + p79 + 79)] p90

p13p34p46p698 + p487 p89 +

p497 + p13p375 p78p89 + p

The steady state probability that the system will be up in the long run is given by

$A_0 = \lim_{t\to\infty} A_0(t) = \lim_{s\to 0} sA_0^*(s) = \lim_{s\to 0} \frac{sN_2(s)}{D_2(s)} =$

$\lim_{s\to 0} N_2(s) \lim_{s\to 0} \frac{s}{D_2(s)}$ As s→ 0, $D_2(s)$ becomes zero.

Therefore, by L' Hospital's rule, A_0 becomes

$A_0 = N_2(0)/D'_2(0)$ **(1.6.5)** where,

D'2 0 = μ0 {p13p34 (p40 + p46p60) p912 + p13p34 + (p13p375 + p12p25) p90 } + (1 − p952) μ1 +
{ μ2p12 + μ3p13 + μ4p13p34 + μ6p13p34p46 + μ5 p952 p13 (1 − 34 (p40 + p46p60) + p1225) + μ7 }
p952 1 − p12p21 − p1334 (p40 + p46p60 + p13p375 + p12p25 1 − p952

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$\mu_8 \{p_{95}^{(2)} p_{78} (1 - p_{12}p_{21}) - p_{13} p_{34} p_{40} (+ p_{46}p_{60} +)] + (1 - p_{95}^{(2)}) [p_{13} (p_{34}p_{48}^{(7)} + p_{37}^{(5)} p_{78})] p_{12}p_{25}p_{78} \} + \mu_9 \{(1 - p_{12}p_{21}) - p_{13} p_{34} p_{40} (+ p_{46}p_{60})\}$

(1.6.6)

Using the results **(1.6.4)** and **(1.6.6)** in **(1.6.5),** we get the expressions for A_0.
The expected up (operative) time of the system during (0,t] is given by

$$\mu_{up}(t) = \int_0^t A_0(u)du$$

so that,

$$\mu^*_{up}(s) = \frac{A^*_0(s)}{s}$$

1.7 Busy Period

a)For Expert Repairman

Let $B_i(t)$ be the probability that the expert repairman is busy in the repair of failed unit at epoch t, when the system initially starts operation from state $S_i \in E$. Solving the resultant set of equations and simplifying for $B_0^{e*}(s)$, we have

$$B_0^{e*}(s) = N_3^e(s) / D_2(s) \qquad \textbf{(1.7.1a)}$$

Where,

$N_3^e(s) = q^*_{01} [(1 - q_{95}^{(2)*} q^*_{57} (q^*_{78}q^*_{89} + q^*_{79})) [q^*_{12}q^*_{25} + q^*_{13}q^*_{34} (q^*_{46}q_{69}^{(8)*} + q_{48}^{(7)*}q^*_{89} + q_{49}^{(7)*}) + q^*_{13}q_{37}^{(5)*}(q^*_{78}q^*_{89} + q^*_{79}) + q^*_{12}q^*_{25}q^*_{57}(q^*_{78}q^*_{89} + q^*_{79})] Z^*_5] + q^*_{01} (1 - (q_{95}^{(2)*} q^*_{57} (q^*_{78}q^*_{89} + q^*_{79}))) q^*_{13} Z^*_3$

In the long run, the expected fraction of time for which the expert server is busy in the repair of failed unit is given by

$$B_0^e = \lim_{t\to\infty} B_0^e(t) = \lim_{s\to 0} s B_0^{e*}(s) = \frac{N_3^e(0)}{D'_2(0)} \qquad \textbf{(1.7.2a)}$$

where

$N_3^e(0) = p_{01}[(1 - p_{95}^{(2)} p_{57} (p_{78}p_{89} + p_{79})) (p_{12}p_{25} + p_{13}p_{34} (p_{46}p_{69}^{(8)} + p_{48}^{(7)}p_{89} + p_{49}^{(7)}) + p_{13}p_{37}^{(5)}(p_{78}p_{89} + p_{79}) + p_{12}p_{25}p_{57} (p_{78}p_{89} + p_{79}))]$ ($(1 - p_{95}^{(2)} p_{57} (p_{78}p_{89} + p_{79}))$

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$p_{13}p_{34}\, p_{46}p_{698} + p_{487}\, p_{89} + p + p_{13}p_{375}\, p_{78}p_{89} + p_{79\,5} + p_{01}\, 1 -$

$p_{952}\, p_{57}\, p_{78}p_{89} + p_{79}\, p_{133}$ **(1.7.3a)** and $D'_2(0)$ is same as given by **(1.6.6)**.

Thus using **(1.7.3a)** and **(1.6.6)**. in **(1.7.2a)**, we get the expression for B_0.

The expected busy period of expert repairman due to repair of failed unit during the time interval (0,t] is given by

$$\mu_b(t) = \int_0^t B_0(u)\,du$$

S

o

$$(s) = \frac{{}_0^{*}(s)}{}$$

that

$*$ B

μ_b s

b)For Ordinary Repairman

Let $B^{o}{}_{i}(t)$ be the probability that the ordinary repairman is busy in the repair of failed unit at epoch t, when the system initially starts operation from state $S_i \in E$. Solving the resultant set of equations and simplifying for $B^{or}{}_0{}^*(s)$, we have

$B^{or}{}_0{}^*(s) = N^{or}{}_4(s)/D_2(s)$ **(1.7.1b)** where,

$N^{or}{}_4(s) = q_{*01}q_{*12}\left(1 - q_{952*}q_{*57}\left(q_{*78}q_{*89} + \right)\right) q_{*79}Z_{*2} + q_{01*}\, q\left(_{*13}q_{34*}\, q_{46*}q_{698*} + q_{487*}q_{89*}\right) + q_{497*} +$

$q_{*13}q_{375*}\, q_{*78}q_{*89} + q_{*79} \quad + q_{*12}q_{*25}q_{*57}q_{*78}q_{*89} + q_{*79}\ _{*9}$

In the long run, the probability that the ordinary repairman will be busy is given by	
Bor0 = lim Bor0 (t) = lim sBor0 * (s) = NDor4'2(0)(0) t→∞s→0 Where	**(1.7.2b)**

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$N_{4r}(0) = p_{12}p_{01}\left[\left(1 - p_{95}^{(2)} p_{57}(p_{78}p_{89} + p_{79})\right)^{2}\right] + p_{01}\left[\left(p_{12}p_{25}p_{57}(p_{78}p_{89} + p_{79}) + \right.\right.$

$p_{13}p_{34}p_{46}p_{69}^{(8)} + p_{48}^{(7)} p_{89} + p_{49}^{(7)} + p_{13}p_{37}^{(5)}p_{78}p_{89} + p_{79}^{(9)}$ **(1.7.3b)**

Then using **(1.7.3b)** and **(1.6.6)** in **(1.7.2b)**, we get the expression for B^{or}_0.

The expected busy period of ordinary repairman during the time interval (0, t] is given by

$\mu_{orb}(t) = \int_0^t B_{or0}(u)\,du$ So that

$\mu_{orb}^{*}(s) = \frac{B_{or0}^{*}(s)}{s}$

1.8) Expected Number of Replacement by Expert Server

Let $V^R(t)$ be the expected number of replacements made by the expert server in (0,t] given that the system starts from the regenerative state S_i at t=0. The solution for $V^R_0(s)$ can be written in the following form:

$V_{R0}(s) = \frac{N_{R5}(s)}{D_2(s)}$ **(1.8.1)**

where,

$N_{R5}(s) = Q_{01}\left(1 - Q^{(2)}_{95} Q_{57}(Q_{89} + Q_{79})\right)\left(Q_{12}Q_{25}Q_{57}Q_{78} + Q_{13}\left(Q_{34}Q_{46} + Q^{(5)}_{37} + \tilde{Q}_{78}\right)\right)\tilde{Q}_{60}\Big] \left[\tilde{Q}_{01}\left(Q_{12}Q_{25}Q_{57} + Q_{13}Q^{(5)}_{37}\right)\left(\tilde{Q}_{78}Q_{89} + \tilde{Q}_{79}\right) + Q_{13}Q_{34}\left(Q_{46}Q^{(8)}_{69} + Q^{(7)}_{48} Q_{89} + Q^{(7)}_{49}\right)\right.$ In steady-state per-unit of time expected number of replacement by expert server is given by

$V_{R0} = \lim_{t\to\infty} \frac{V_{0R}(t)}{t} = \lim_{s\to 0} s\tilde{V}_{0R}(s) = \frac{N_{R5}(0)}{D_2'(0)}$ **(1.8.2)**

$N_{R5} = \left[p_{01}\left(1 - p^{(2)}_{95} p_{57}(p_{78}p_{89} + p_{79})\right)\left(p_{12}p_{25}p_{57}p_{78} + p_{13}\left(p_{34}p_{46} + p^{(5)}_{37} p_{78}\right)\right) + p_{01}\left(p_{12}p_{25}p_{57} + p_{13}p^{(5)}_{37}\right)\left(p_{78}p_{89} + p_{79}\right) + p_{13}p_{34}\left(p_{46}p^{(8)}_{69} + p^{(7)}_{48} p_{89} + p^{(7)}_{49}\right)p_{90}\right]$

(1.8.3)

Here we have used $Q_{ij}(0) = p_{ij}$

Thus using **(1.8.3)** and **(1.6.6)**. in **(1.8.2),** we get the expression for V^R_0.

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a) Expected Number of Repairs by Expert Repairman

Let V^e_i (t)be the expected number of repairs by the expert repairman in (0,t] given that the system starts from the regenerative state S_i at t=0. The solution for $V^e_0(s)$ can be written in the following form:

Ve( )s= ND26e((ss)) **(1.8.1a)**

where,

$N^e_6(s) = (Q_{12}Q_{25} + Q_{13}Q_{34})(1 - Q^{(2)}_{95})$
$+ Q_{13}Q_{95}^{(2)}[$
$Q_{34}Q_{46}Q^{(8)}_{69} + Q_{34}Q_{48} + Q_{34}Q^{(7)}_{49} +$ **(1.8.2a)**
$Q_{12}Q_{25}Q^{(5)}_{37}Q_{78} + Q^{(5)}_{37}Q_{79}]$

Insteady-state perunit of time expected number of repairs by expert repairman is given by

$V_0 = \lim_{t\to\infty} \frac{V^e_0(t)}{t} = \lim_{s\to 0} s\tilde{V}^e_0(s) = \frac{N^e_6(0)}{D_2(0)}$ **(1.8.2a)** **(1.8.2a)**

$N^e_6(0) = (p_{12}p_{25} + p_{13}p_{34})\left(1 - p^{(2)}_{95}\right) + p_{13}p^{(2)}_{95}[p_{34}p_{46}p^{(8)}_{69} + p_{34}p_{48} + p_{34}p^{(7)}_{49} + p_{12}p_{25}p^{(5)}_{37}p_{78} + p^{(5)}_{37}p_{79}]$ **(1.8.3a)**

Here we have used $\tilde{Q}_{ij}(0) = p_{ij}$

Thus using **(1.8.3a)** and **(1.6.6)**. in **(1.8.2a),** we get the expression for V^e_0

b) Expected Number of Repairs By Ordinary Repairman

Let V^o_i (t)be the expected number of repairs by the ordinary repairman in (0,t] given that the system starts from the regenerative state S_i at t=0. In steady-state per-unit of time expected number of repairs by ordinary repairman is given by

$V^o_0 = \lim_{t\to\infty} \frac{V^o_0(t)}{t} = \lim_{s\to 0} s\tilde{V}^o_0(s) = \frac{N^o_7(0)}{D_2(0)}$ **(1.8.1b)**

$N^o_7(0) = \left(1 - p^{(2)}_{95}\right)\left[p_{12} + p_{13}p^{(5)}_{37} + p_{13}p_{34}\left(p_{46}p^{(8)}_{69} + p^{(7)}_{48} + p^{(7)}_{49} + p^{( )}_{90} + p^{( )}_{912}\right)\right]$ **(1.8.2b)**

Here we have used $\tilde{Q}_{ij}(0) = p_{ij}$

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Thus using **(1.8.2b)** and**(1.6.6)** in **(1.8.1b),** we get the expression for V_0^o.

1.9.Profit Function Analysis

Two profit functions $P_1(t)$ and $P_2(t)$ can be easily obtained for the system model under study with the help of characteristics obtained earlier. The expected total profits incurred during (0,t] are:

$P_1(t)$ = Expected total revenue in (0, t] − Expected total expenditure in (0, t]

$= K_0\mu_{up}(t) - K_1\mu_{eb}(t) - K_2\mu_{ob}(t)$ **(1)**

Similarly,

$P_2(t) = K_0\mu_{up}(t) - K_3V_0^{rp}(t) - K_4V_0^{e}(t) - K_5V_{o0}(t)$ **(2)** where,

K_0 is revenue per unit up time of the system.

K_1 is repair cost per unit of time by expert repairman.

K_2 is repair cost per unit of time by ordinary repairman.

$K_3$is per unit repalcement cost of the failed unit. K_4 is per unit repair cost by expert repairman.

K_5 is per unit repair cost by ordinary repairman.

Now the expected total profits per unit time, in steady state, is given by

$$P_1 = \lim_{t\to\infty} \frac{P_1(t)}{t} = \lim_{s\to 0} s^2 P_1^*(s)$$

and

$$P_2 = \lim_{t\to\infty} \frac{P_2(t)}{t} = \lim_{s\to 0} s^2 P_2^*(s)$$

so that

$P_1 = K_0A_0 - K_1B_{e0} - K_2B_{o0}$ **and**	**(3)**
$P_2 = K_0A_0 - K_3V_{re0} - K_4V_{e0} - K_5V_{o0}$	**(4)**

1.10 Conclusion

To study the behavior of MTSF, Availability and profit function through graphs w.r.t various parameters, we plot curves for these three characteristics w.r.t failure parameter $_1$ in Fig.1.2 and 1.3 and 1.4 respectively for three

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different values of repair rate $_3$ = (0.60,0.70,0.80) whereas other parameters are kept fixed as $\alpha_2 = 0.75$, $\gamma_1 = 0.30$, $\gamma_2 = 0.20$, $\beta_1 = 0.20$, $\beta_2 = 0.45$, p = 0.5, q = 0.5.

Fig 1.2 indicates that the graph for MTSF decreases with the increase in the failure rate $_1$ and increases with the increase in repair rate $_3$.

Fig 1.3 clearly shows that the graph for Availability decreases with the increase in the failure rate $_1$ and increases with the increase in repair rate $_3$.

Similarly Fig 1.4 shows that the graph for profit function decreases with the increase in failure rate $_1$ and increases with the increase in repair rate $_3$.

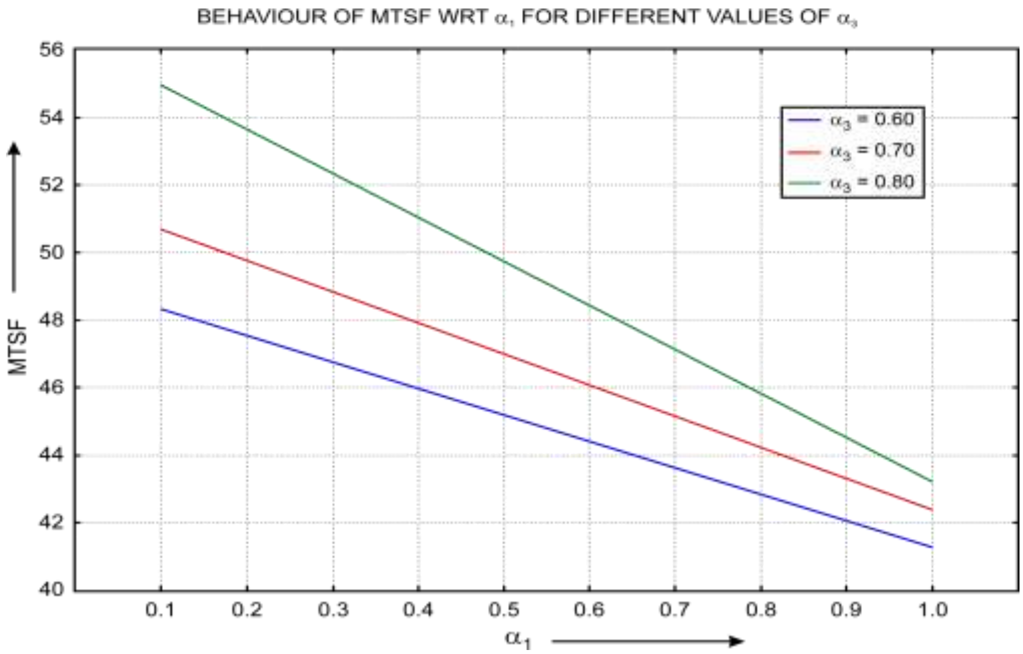

Fig 1.2

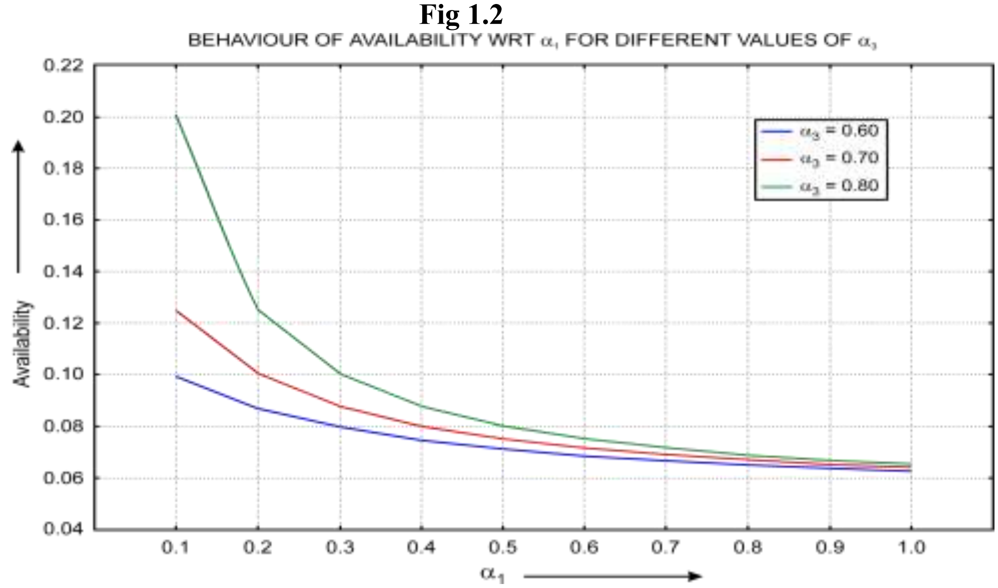

FIG 1.3

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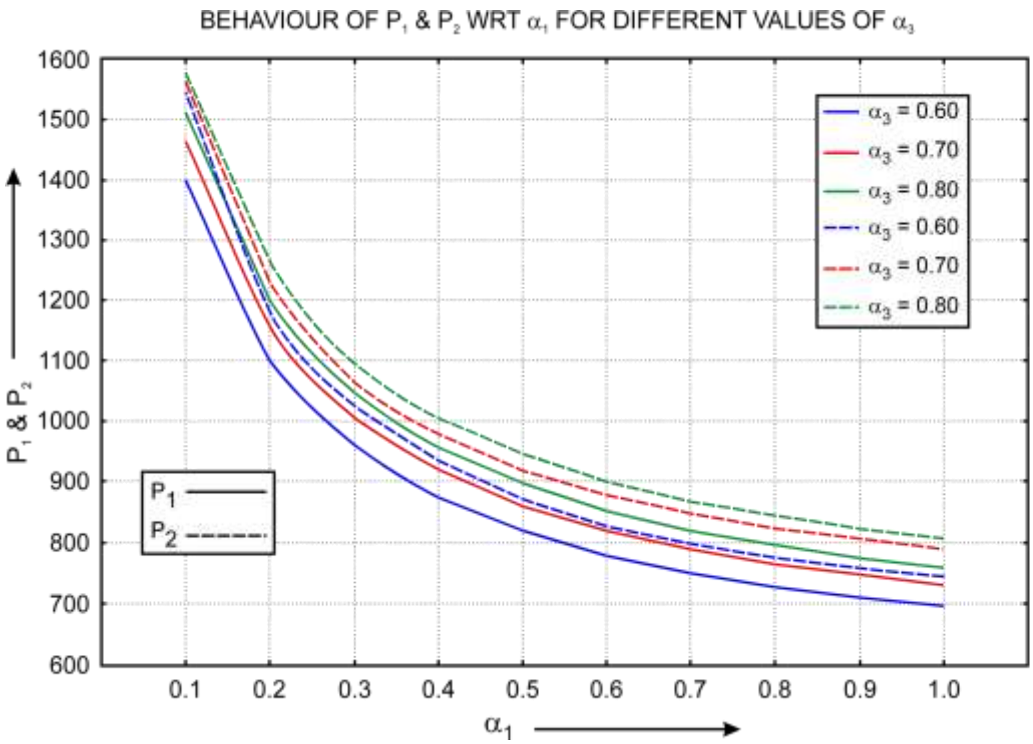

FIG 1.4

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