A Clear Step-by-Step Revised Simplex Method

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Abstract
The revised simplex method is a central algorithm in linear programming for solving optimization problems efficiently. This paper offers a detailed, step-by-step exposition of its procedure tailored for learners and practitioners. Without performing comparative analyses, we focus on explaining each computational stage clearly. We illustrate the method through two (2) numerical examples to demonstrate its practical utility. The result is an accessible guide that bridges theorem and implementation, empowering readers to apply the method themselves.

Keywords: Linear programming, revised simplex method, simplex tableau, computational procedure, mathematical modeling.

1Introduction
Linear programming is a fundamental tool in optimization leveraged for modeling decision-making in operations research, logistics, economics, engineering and business context. Traditional algorithms such as the simplex method remain foundational, but direct tableau updates grow burdensome for large systems. The Revised Simplex Method addresses this by updating only selected matrix components – primarily the basis inverse and relevant vectors – thus lowering memory and computational demands. This work does not aim to contrast the Revised Simplex with other methods; rather, it seeks to explain its step-by-step implementation. By working through two representative examples, we demonstrate how the method operates in practice.

2.Materials and Methods
To solve linear programming problems using the Revised Simplex Method, a structured sequence of steps is followed to systematically reach the optimal solution. The method begins by transforming the problem into a standard mathematical form and then proceeds through iterative calculations that update the solution until optimality is achieved.

2.1Formulation of Linear Programming Problems
Formulation of linear programming problems (LPP) involves translating a real-life problem into a mathematical model into three (3) main components, namely:
i.Decision Variables: These are the unknowns that represent the choices available.
ii.Objective Function: This is the function to be maximized or minimized
iii.Constraints: These are the limitations or requirements of the problems, expressed as linear inequalities or equations.
We consider linear programs in standard form:
$maximize\ Z = c^T X$, subject to
$AX = b, X \geq 0$
Where $A$is an mxn matrix, $X \in R^n$, $b \in R^m$ and $c \in R^n$

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3.Algorithm Steps:

The procedure is organized into nine (9) clear steps;

Step one (1): Convert the linear programming problem (LPP) into standard form

Standard form I: In this form, it is assumed that an identity matrix is obtained after introducing slack variables only.

Standard form II: If artificial variables are needed for an identity matrix, then two-phase method of ordinary simplex method is used in a slightly different way to handle artificial variables.

Converting to Standard Form

Case I: Maximization Type of Problems with ≤ Type of Constraints

The real world problem can be formulated as;

$maximze\ Z = c_1x_1 + c_2x_2 + \cdots + c_nx_n$

Subject to the constraints

$$\begin{array}{llll} a_{11}x_1 + & a_{12}x_2 + & \ldots + a_{1n}x_n \le b_1 \\ a_{21}x_1 + & a_{22}x_2 + & \ldots + a_{2n}x_n \le b_2 \\ \vdots & \vdots & \vdots \\ a_{m1}x_1 + & a_{m2}x_2 + & \ldots + a_{mn}x_n \le b_m \end{array}$$

$x_1, x_2, \ldots, x_n \ge 0$

If the objective function is to be minimized, then convert the given problem into a problem of maximization by minimizing $Z = maximize\ (-Z)$. Check whether all the b_i are positive; if any one of the $b_i < 0$, then multiply both sides of that constraint by (-1) so as to get all $b_i \ge 0$. Add slack variables in the left hand side of constraint and assign a zero coefficient to these in the objective function (Z).

Then write the problem in standard form as

$$maximze\ Z = c_1x_1 + c_2x_2 + \cdots + c_nx_n + 0s_1 + 0s_2 + \cdots + 0s_n$$

subject to the constraints

$$\begin{array}{llll} a_{11}x_1 + & a_{12}x_2 + & \ldots + a_{1n}x_n + s_1 = b_1 \\ a_{21}x_1 + & a_{22}x_2 + & \ldots + a_{2n}x_n + s_2 = b_2 \\ \vdots & \vdots & \vdots \\ a_{m1}x_1 + & a_{m2}x_2 + & \ldots + a_{mn}x_n + s_m = b_m \end{array}$$

and we find the initial basic feasible solution by substituting the values of the decision variables
equals zero i.e $x_1 = x_2 = \cdots = x_n = 0$ and obtain $s_1 = b_1$, $s_2 = b_2 \ldots, s_m = b_m$

Case II: Minimization Problems with constraints ≥ type

Here, we write in standard form as:

$maximze\ Z = c_1x_1 + c_2x_2 + \cdots + c_nx_n + 0s_1 + 0s_2 + \cdots + 0s_n$

subject to the constraints

$$\begin{array}{llll} a_{11}x_1 + & a_{12}x_2 + & \ldots + a_{1n}x_n - s_1 = b_1 \\ a_{21}x_1 + & a_{22}x_2 + & \ldots + a_{2n}x_n - s_2 = b_2 \\ \vdots & \vdots & \vdots \\ a_{m1}x_1 + & a_{m2}x_2 + & \ldots + a_{mn}x_n - s_m = b_m \end{array}$$

Where $x_i, s_i \ge 0$

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The initial basic solution is gotten by putting $x_1 = 0$, $x_2 = 0$ ... and we obtain $s_1 = -b_1$, $s_2 = -b_2$, $s_3 = b_3$, ... which clearly violates non-negativity constraints ($s_j \geq 0$), hence it is not feasible. $'m'$ new variables called Artificial variables $(A_1, A_2, A_3 \ldots, A_m)$ are added to left hand side (one on each equations) as

$$\begin{array}{cccc} a_{11}x_1 + & a_{12}x_2 + & \ldots + a_{1n}x_n - s_1 + A_1 = b_1 \\ a_{21}x_1 + & a_{22}x_2 + & \ldots + a_{2n}x_n - s_2 + A_2 = b_2 \\ \vdots & \vdots & \vdots \\ a_{m1}x_1 + & a_{m2}x_2 + & \ldots + a_{mn}x_n - s_m + A_m = b_m \end{array}$$

x_i, s_j and $A_j \geq 0$

Case III: Minimization Type of LPP with mixed constraints

Example: Minimize $Z = ax_1 + bx_2$
subject to:
$x_1 + x_2 = c_1$
$x_1 \leq c_2$
$x_2 \geq c_3$
$x_1, x_2 \geq 0$
Then the problem can be be converted to standard form as follows:
Minimize $Z = ax_1 + bx_2 + 0s_1 + 0s_2 + MA_1 + MA_2$
Subject to:
$x_1 + x_2 + A_1 = c_1$
$x_1 + s_1 \leq c_2$
$x_2 - s_2 + A_1 \geq c_3$
and the initial basic feasible solution ($IBFS$) is given by:
$x_1, x_2 = 0$
$A_1 = c_1$, $s_1 = c_2$, $A_2 = c_3$ $s_2 = 0$)

Case IV: Maximization Type of LPP with mixed constraints

Maximize $Z = 4x_1 + 5x_2 - 3x_3$
Subject to:
$x_1 + x_2 + x_3 = 10$
$x_1 - x_2 \geq 7$
$3x_1 + 5x_2 + 4x_3 \leq 58$
Then we can convert to standard form as:
Maximize $Z = 4x_1 + 5x_2 - 3x_3 + 0s_1 + 0s_2 - MA_1 - MA_2$
Subject to:
$x_1 + x_2 + x_3 + A_1 = 10$
$x_1 - x_2 - s_1 + A_2 = 7$
$3x_1 + 5x_2 + 4x_3 + s_2 = 58$
And $IBFFS$ is given by
$x_1, x_2, x_1 = 0$ $A_1 = 10, A_2 = 7, s_2 = 58$

Case V: Unbounded Solution

Example: Maximize $Z = 4x_1 + 5x_2 - 3x_3 + 5x_4$
Subject to:
$4x_1 - 6x_2 - 5x_3 - 4x_4 \geq -20$
$-3x_1 - 2x_2 + 4x_3 + x_4 \leq 5$
$-8x_1 - 3x_2 + 3x_3 + 2x_4 \leq 50$
$x_1, x_2, x_3, x_4 \geq 0$
The problem can be converted to standard form as:
Multiply first constraints by -1 and write in standard form

$$Z = 4x_1 + 5x_2 - 3x_3 + 5x_4 + 0s_1 + 0s_2 + 0s_3$$

$$\begin{array}{c} 4x_1 - 6x_2 - 5x_3 - 4x_4 + s_1 = -20 \\ -3x_1 - 2x_2 + 4x_3 + x_4 + s_2 = 5 \\ -8x_1 - 3x_2 + 3x_3 + 2x_4 + s_3 = 50 \end{array}$$

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And the initial basic feasible solution is gotten by putting $x_1 = x_2 = x_3 = x_4 = 0$ and obtain $s_1,\ s_1 = 20,\ s_2 = 5,\ s_3 = 50,$ and $z = 0$.

Step Two (2): Find the initial basis feasible solution with initial basis
B = IM (IM =Identity matrix)

Step Three (3): Consider the objective function $Z = CX$ subject to $AX = b$ and find the value of $A, b, C, \hat{A}$ and $\hat{b}$. Where
$\hat{A} = \begin{pmatrix} A \\ -C \end{pmatrix}$ and $\hat{b} = \begin{pmatrix} b \\ 0 \end{pmatrix}$

Step Four (4): Find the value of $\hat{B}^{-1}$ ($B -$ Cap inverse) where;
$\hat{B}^{-1} = \begin{pmatrix} B^{-1} & 0 \\ C_B B^{-1} & 1 \end{pmatrix}$

Step Five (5): Compute the net evaluation
$Z_j - C_j = (C_B B^{-1} \quad 1)\hat{A}$
Where $(C_B B^{-1} \quad 1) \rightarrow$ last row of $\hat{B}^{-1}$

Case I: If all $Z_j - C_j \geq 0$, then current solution is an optimal solution. So find $\hat{X}_B$

Case II: If atleast one $Z_j - C_j < 0$, find the most negative of them say $Z_K - C_K$ corresponding to the variable X_K enters the basis.

Step Six (6): We compute
$$\hat{X}_K = \hat{B}^{-1}\hat{a}_K$$

Case I: If all $\hat{X}_K \leq 0$, then there exist an unbounded solution to the LPP

Case II: If atleast one $\hat{X}_K > 0$, then find the value of $\hat{X}_B = \hat{B}^{-1}\hat{b}$

Step Seven (7): Write down the results in the revised simplex table and find the minimum of
$\left\{\frac{\hat{X}_B}{\hat{X}_K}, X_K > 0\right\}$
and also find the key element.

Step Eight (8): Convert the key element unity and all other elements of the key column to zero and improve the value of $\hat{B}^{-1}$

Step Nine (9): Go to step Five (5) and repeat the procedure until an optimal feasible solution is obtained or there is an indication of an unbounded solution.

4. Results and Discussion

Two numerical examples are presented to illustrate the Revised Simplex Method and demonstrate each computational step clearly.

Each example is solved step-wise, highlighting the critical operations involved in reaching the optimal solution. The examples of increasing complexity illustrates the method's application.

Example 1

Use the revised simplex method to solve the LPP

$$Max\ Z = 6x_1 - 2x_2 + 3x_3$$
$$\text{subject to} \quad \begin{aligned} 2x_1 - x_2 + 2x_3 &\leq 2 \\ x_1 + 4x_3 &\leq 4 \\ x_1, x_2, x_3 &\geq 0 \end{aligned}$$

Solution

Step One (1): We convert the given problem into its standard form by adding slack variables s_1 and s_2

$$Max\ Z = 6x_1 - 2x_2 + 3x_3 + 0s_1 + 0s_2$$
$$\text{subject to} \quad \begin{aligned} 2x_1 - x_2 + 2x_3 + s_1 &= 2 \\ x_1 + 4x_3 + s_2 &= 4 \\ x_1, x_2, x_3, s_1, s_2 &\geq 0 \end{aligned}$$

Step Two (2): The initial basis feasible solution with initial basis $B = IM$
$x_1 = x_2, = x_3 = 0, s_1 = 2, s_2 = 4$

Step Three (3):

$$A = \begin{array}{c} \begin{matrix} x_1 & x_2 & x_3 & s_1 & s_2 \end{matrix} \\ \begin{pmatrix} 2 & -1 & 2 & 1 & 0 \\ 1 & 0 & 4 & 0 & 1 \end{pmatrix} \end{array}, \boldsymbol{b} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}, c = (6 \quad -2 \quad 3 \quad 0 \quad 0)$$
(Coefficient of the objective function.

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$$\hat{A} = \begin{pmatrix} A \\ -c \end{pmatrix} = \begin{pmatrix} a_1 & a_2 & a_3 & a_4 & a_5 \\ x_1 & x_2 & x_3 & x_4 & x_5 \\ 2 & -1 & 2 & 1 & 0 \\ 1 & 0 & 4 & 0 & 1 \\ -6 & 2 & -3 & 0 & 1 \end{pmatrix}, \hat{b} = \begin{pmatrix} b \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \\ 0 \end{pmatrix}$$

Step Four (4):

$B = (s_1 \quad s_2) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = B^{-1}$

$c_B = (0 \quad 0)$ (Coefficient of basis in the objective function)

$c_B B^{-1} = (0 \quad 0) \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = (0 \quad 0)$

$\hat{B} = \begin{pmatrix} B^{-1} & 0 \\ c_B B^{-1} & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

Step Five (5): We compute the net evaluation

$z_j - c_j = (c_B B^{-1} \quad 1)\hat{A}$

$= (0 \quad 0 \quad 1) \begin{pmatrix} 2 & -1 & 2 & 1 & 0 \\ 1 & 0 & 4 & 0 & 1 \\ -6 & 2 & -3 & 0 & 0 \end{pmatrix} = (-6 \quad 2 \quad -3 \quad 0 \quad 0)$

$z_1 - c_1 = -6$ most negative, so x_1 enters the basis

Step Six (6):

$\hat{x}_1 = \hat{B}^{-1}\hat{a}_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \\ -6 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ -6 \end{pmatrix}$

$\hat{x}_B = \hat{B}^{-1}\hat{b} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 2 \\ 4 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \\ 0 \end{pmatrix}$

Step Seven (7): Revised simplex table is shown as

B	$\hat{x}_B$	$\hat{B}^{-1}$	$\hat{x}_1$	**Ratio** $\frac{\hat{x}_B}{\hat{x}_1}$
s_1	**2**	**1 0 0**	**(2)**	$2/2 = 1$
s_2	**4**	**0 1 0**	**1**	$4/\underline{1} = 4$
Z	**0**	**0 0 1**	-6	

2 is the key element and therefore s_1, leaves the basis.

Step Eight (8):

$$\hat{B}^{-1}_{(Current)} = \begin{pmatrix} \frac{1}{2} & 0 & 0 \\ -\frac{1}{2} & 1 & 0 \\ 3 & 0 & 1 \end{pmatrix} \begin{matrix} R_1^1, R_1^1 \to \frac{R_1}{2} \\ R_2^1, R_2^1 \to R_2 - R_1^1 \\ R_3^1, R_3^1 \to R_3 + 6R_1^1 \end{matrix}$$

Step Nine (9): Go to step Five (5)

$z_j - c_j = (c_B B^{-1} \quad 1)\hat{A}$

$= (3 \quad 0 \quad 1) \begin{pmatrix} 2 & -1 & 2 & 1 & 0 \\ 1 & 0 & 4 & 0 & 1 \\ -6 & 2 & -3 & 0 & 0 \end{pmatrix} = (0 \quad -1 \quad 3 \quad 3 \quad 0)$

$z_2 - c_2 = -1$, the most negative, x_2 enters the basis

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$$\hat{x}_2 = \hat{B}^{-1}\hat{a}_2 = \begin{pmatrix} -\frac{1}{2} & 0 & 0 \\ \frac{1}{2} & 1 & 0 \\ 3 & 0 & 1 \end{pmatrix}\begin{pmatrix} -1 \\ 0 \\ 2 \end{pmatrix} = \begin{pmatrix} -\frac{1}{2} \\ \frac{1}{2} \\ -1 \end{pmatrix}$$

$$\hat{x}_B = \hat{B}^{-1}\hat{b} = \begin{pmatrix} \frac{1}{2} & 0 & 0 \\ -\frac{1}{2} & 1 & 0 \\ 3 & 0 & 1 \end{pmatrix}\begin{pmatrix} 2 \\ 4 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \\ 6 \end{pmatrix}$$

	B	$\hat{x}_B$	$\hat{B}^{-1}$	$\hat{x}_2$	Ratio ${}^{\hat{x}_B}/_{\hat{x}_2}, \hat{x}_2 > 0$
R_1 R_2 R_3	x_1 s_2 Z	**1** **3** **6**	$\frac{1}{2}$ **0 0** $-\frac{1}{2}$ **1 0** **3 0 1**	$-\frac{1}{2}$ $\left(\frac{1}{2}\right)$ -1	$-$ (6) min $-$

s_2 leaves the basis $\left(\frac{1}{2}\right)$ is the key element.
Next we make the key element $\frac{1}{2}$ to be unity and other elements in the key column to be zero.

$$\hat{B}^{-1} = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 2 & 0 \\ 2 & 2 & 1 \end{pmatrix} \quad \begin{matrix} R_1^1, R_1^1 \to \frac{R_2^1}{2} \\ R_2^1, R_2^1 \to \frac{R_2}{1/2} \\ R_3^1, R_3^1 \to R_3 + R_2^1 \end{matrix}$$

$$\mathbf{z_j - c_j = (c_B B^{-1} \quad 1)\hat{A}}$$

$$= (2 \quad 2 \quad 1)\begin{pmatrix} 2 & -1 & 2 & 1 & 0 \\ 1 & 0 & 4 & 0 & 1 \\ -6 & 2 & -3 & 0 & 0 \end{pmatrix} = (0 \quad 0 \quad 9 \quad 2 \quad 2)$$

Since all $z_j - c_j > 0$, the current feasible solution is optimal

$$\hat{x}_B = \hat{B}^{-1}\hat{b} = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 2 & 0 \\ 2 & 2 & 1 \end{pmatrix}\begin{pmatrix} 2 \\ 4 \\ 0 \end{pmatrix} = \begin{pmatrix} 4 \\ 6 \\ 12 \end{pmatrix}$$

$\therefore$ the optimal solution is given by $x_1 = 4$, $x_2 = 6$, and $Z = 12$

Example 2
Use the revised simplex method to solve the following LPP
$Max\ Z = x_1 + 2x_2$

Subject to:
$x_1 + x_2 \leq 3$
$x_1 + 2x_2 \leq 5$
$3x_1 + x_2 \leq 6$
$x_1, x_2 \geq 0$

Solution

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Step One (1): Introduce the slack variables into its standard form $s_1,\ s_2,\ s_3$ and convert the given problem

$$Max\ Z = x_1 + 2x_2 + 0s_1 + 0s_2 + 0s_3$$
$$\text{Subject to: } \begin{aligned} x_1 + x_2 + s_1 &= 3 \\ x_1 + 2x_2 + s_2 &= 5 \\ 3x_1 + x_2 + s_3 &= 6 \end{aligned}$$
$$x_1, x_{2,} s_1, \quad s_2, \quad s_3 \geq 0$$

Step Two (2): The initial basis feasible solution is
$x_1 = x_2 = 0,\ s_1 = 3,\ s_2 = 5,\ s_3 = 6$

Step Three (3):

$$\mathbf{A} = \begin{pmatrix} \mathbf{1} & \mathbf{1} & \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{1} & \mathbf{2} & \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{3} & \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{1} \end{pmatrix}, \mathbf{b} = \begin{pmatrix} \mathbf{3} \\ \mathbf{5} \\ \mathbf{6} \end{pmatrix}, \mathbf{c} = (\mathbf{1} \quad \mathbf{2} \quad \mathbf{0} \quad \mathbf{0} \quad \mathbf{0})$$

$$\widehat{\mathbf{A}} = \begin{pmatrix} \mathbf{A} \\ -\mathbf{c} \end{pmatrix} = \begin{pmatrix} \mathbf{1} & \mathbf{1} & \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{1} & \mathbf{2} & \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{3} & \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{1} \\ -\mathbf{1} & -\mathbf{2} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{pmatrix}, \hat{\mathbf{b}} = \begin{pmatrix} \mathbf{b} \\ \mathbf{0} \end{pmatrix} = \begin{pmatrix} \mathbf{3} \\ \mathbf{5} \\ \mathbf{6} \\ \mathbf{0} \end{pmatrix}$$

Step Four (4):

$$\boldsymbol{B} = (\boldsymbol{s_1} \quad \boldsymbol{s_2} \quad \boldsymbol{s_3}) = \begin{pmatrix} \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} \end{pmatrix} \textbf{ and } \boldsymbol{B}^{-1} = \begin{pmatrix} \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} \end{pmatrix}$$

$\boldsymbol{c_B} = (\mathbf{0} \quad \mathbf{0} \quad \mathbf{0})$ **(Coefficient of basis in the objective function)**

$$c_B B^{-1} = (0 \quad 0 \quad 0) \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = (0 \quad 0 \quad 0)$$

$$\hat{B} = \begin{pmatrix} B^{-1} & 0 \\ c_B B^{-1} & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Step Five (5): The net evaluations are given by

$$\mathbf{z_j} - \mathbf{c_j} = (\mathbf{c_B B^{-1}} \quad \mathbf{1})\widehat{\mathbf{A}}$$

$$= (\mathbf{0} \quad \mathbf{0} \quad \mathbf{0} \quad \mathbf{1}) \begin{pmatrix} \mathbf{1} & \mathbf{1} & \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{1} & \mathbf{2} & \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{3} & \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{1} \\ -\mathbf{1} & -\mathbf{2} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{pmatrix} = (-\mathbf{1} \quad -\mathbf{2} \quad \mathbf{0} \quad \mathbf{0} \quad \mathbf{0})$$

$$\boldsymbol{z_1} - \boldsymbol{c_1} = -\mathbf{2}$$

Thus, the most negative value is -2, and x_2 enters the basis

Step Six (6):

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$$\hat{\mathbf{x}}_2 = \hat{\mathbf{B}}^{-1}\hat{\mathbf{b}}_1 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 1 \\ -2 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 1 \\ -2 \end{pmatrix}$$

$$\hat{\mathbf{x}}_B = \hat{\mathbf{B}}^{-1}\hat{\mathbf{b}} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 3 \\ 5 \\ 6 \\ 0 \end{pmatrix} = \begin{pmatrix} 3 \\ 5 \\ 6 \\ 0 \end{pmatrix}$$

Step Seven (7): Revised simplex table is shown as

B	$\hat{\mathbf{x}}_B$	$\hat{\mathbf{B}}^{-1}$	$\hat{\mathbf{x}}_2$	Ratio $\frac{\hat{\mathbf{x}}_B}{\hat{\mathbf{x}}_2}$
$\mathbf{s}_1$	3	1 0 0 0	1	${}^{3}/_{1} = 1$
$\mathbf{s}_2$	5	0 1 0 0	(2)	${}^{5}/_{2} = (2.5)$ min
$\mathbf{s}_3$	6	0 0 1 0	1	${}^{6}/_{1} = 6$
Z	0	0 0 0 1	−2	

s_2, leaves the basis.

Step Eight (8): First iteration
Drop s_2 and introduce x_2. Convert the leading element into '1' and the remaining element as zero in the key column

$$\hat{\mathbf{B}}^{-1} = \begin{pmatrix} 1 & -\frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 \\ 0 & -\frac{1}{2} & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix} \mathbf{R}_1 - \mathbf{R}_2 \to (1 \quad 0 \quad 0 \quad 0 \quad 1) - \left(0 \quad \frac{1}{2} \quad 0 \quad 0 \quad 1\right)$$

$$= \left(1 \quad -\frac{1}{2} \quad 0 \quad 0 \quad 0\right)$$

$c_B B^{-1} = (0 \quad 1 \quad 0 \quad 1)$ The last row of $\hat{B}^{-1}$
Compute $z_j - c_j = (c_B B^{-1} \quad 1)\hat{A}$

$$= (0 \quad 1 \quad 0 \quad 1) \begin{pmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 2 & 0 & 1 & 0 \\ 3 & 1 & 0 & 0 & 0 \\ -1 & -2 & 0 & 0 & 0 \end{pmatrix} = (0 \quad 0 \quad 0 \quad 1 \quad 0)$$

$\therefore \; z_j - c_j \geq 0$
Since all $z_j - c_j \geq 0$, we have reached the optimal solution

$$\hat{x}_B = \hat{B}^{-1}\hat{b} = \begin{pmatrix} 1 & -\frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 \\ 0 & -\frac{1}{2} & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 3 \\ 5 \\ 6 \\ 0 \end{pmatrix} = \begin{array}{c} \begin{pmatrix} \frac{1}{2} \\ \frac{5}{2} \\ \frac{7}{2} \\ 5 \end{pmatrix} \end{array} \begin{array}{l} s_1 \\ s_2 \\ s_3 \\ Z \end{array}$$

$\therefore$ the optimal solution is given by $x_1 = 0$, $x_2 = \frac{5}{2}$, and $Max\ Z = 12$

5.Discussion

The Revised Simplex Method stands out as an efficient computational alternative to the classical simplex approach by limiting updates to only essential components of the tableau. Rather than recalculating the entire tableau at each iteration, the method focuses on updating the basis inverse and relevant vectors, significantly reducing both memory consumption and arithmetic operations. This makes it particularly suitable for solving large-scale and sparse linear programming problems where computational cost is a major concern.

The results from the two illustrative examples clearly demonstrate the algorithm's systematic nature and operational efficiency. Each pivot iteration refines the feasible region by updating basic and non-basic variables while ensuring that feasibility and optimality conditions are preserved. The smooth progression from an initial feasible solution to the optimal one highlights the precision and consistency of the Revised Simplex approach.

A key advantage observed is the method's adaptability to computer implementation. Its reliance on structured matrix algebra aligns well with modern programming environments such as Python, MATLAB, and R, where matrix operations can be optimized using built-in linear algebra libraries. Consequently, the Revised Simplex Method is not only pedagogically valuable for classroom demonstrations but also highly relevant for developing computational solvers and decision-support systems.

Moreover, the algorithm's modular structure enhances its scalability and maintainability, making it a foundation for advanced optimization techniques such as the Dual Revised Simplex Method, Interior-Point Algorithms, and Sparse Matrix Variants. This highlights its enduring importance in both theoretical research and applied operations research, where efficiency and clarity of computation remain essential.

6.Contribution of the Study

This paper contributes to the body of knowledge in optimization and linear programming by presenting a clearly structured and pedagogically enhanced exposition of the Revised Simplex Method. Unlike most existing works that emphasize theoretical derivations or algorithmic comparisons, this study focuses on simplifying and clarifying the computational process for learners, instructors, and practitioners.

The work's foremost contribution lies in its step-by-step procedural framework, which divides the Revised Simplex Method into nine distinct and logically connected stages. This structured presentation improves conceptual understanding and practical application, making the method more accessible for instructional use and manual computation.

Another contribution is the inclusion of comprehensive numerical examples that illustrate the transition from theoretical

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formulation to optimal solutions. These examples serve as effective learning tools, bridging the gap between mathematical modeling and implementation.

Additionally, the paper highlights the algorithm's compatibility with modern computational environments such as Python, MATLAB, and R, emphasizing its potential for integration into educational and decision-support software. This aligns the classical Revised Simplex Method with contemporary computational needs and promotes its ongoing relevance in research, industry, and academic instruction.

7.Conclusion

This study presented a clear, structured, and pedagogically focused exposition of the Revised Simplex Method for solving linear programming problems. By detailing each computational step, the paper bridges the gap between theoretical formulation and practical application, providing both learners and practitioners with a transparent view of the algorithm's mechanics.

The method's efficiency arises from its selective matrix updates, which minimize redundant computations while maintaining feasibility and optimality. The worked examples demonstrated the algorithm's logical progression and confirmed its capacity to yield accurate solutions through systematic pivot operations.

Beyond its instructional value, the Revised Simplex Method remains a powerful computational tool for real-world optimization. Its structure lends itself naturally to computer implementation using modern numerical platforms, enabling efficient solutions for high-dimensional and data-intensive problems.

Future research can extend this work by integrating the Revised Simplex Method into educational and industrial software, exploring its dual or parametric variants, and applying it to complex optimization domains such as network flows, transportation, and resource allocation. These directions would not only strengthen its practical relevance but also reinforce its central role in the advancement of modern optimization theory and applications.

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