Hybrid Optimization Approaches: A Systematic Review of the Whale Optimization Algorithm and its Variants

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Abstract- In this paper, we proposed a hybrid approach that combines the Whale Optimization Algorithm (WOA) with the Particle Swarm Optimization (PSO) algorithm. Using this hybridization, this paper achieved a balance between exploration and exploitation. By combining the strengths of both algorithms, this hybrid approach enhances solution accuracy and convergence speed. To evaluate its effectiveness, this paper tested the hybrid model using a total of 23 benchmark functions, which provided insights into its performance across various optimization scenarios. This review explores WOA and its hybrid variations, discussing their improvements.

Keywords- Benchmarks, Optimization, Hybridization, Algorithm, Whale Optimization Algorithm, Particle Swarm Optimization

1. Introduction

Whales are the biggest mammals in the earth. The Whale Optimization Algorithm (WOA) is a nature-inspired metaheuristic algorithm. There are many techniques to improve The Whale Optimization Algorithm performance, hybridization has emerged as a promising approach. In this paper, we proposed a new method that combines two well-known and advanced optimization algorithms that are the Whale Optimization Algorithm (WOA) and the Particle Swarm Optimization (PSO) algorithm. WOA is great at exploration which means that it searches widely across different possible solutions, while PSO is excellent at exploitation which means that it focuses on improving the current best solution. By bringing them together, our hybrid approach balances both exploration and exploitation, helping to find better solutions faster and more efficiently. To test how well this hybrid method works, we used 23 benchmark functions that cover different types of optimization problems. These tests helped us understand the strengths of our approach in finding the best solutions. These outcomes showed that our hybrid method performs excellent than using Whale Optimization Algorithm (WOA) or Particle Swarm Optimization (PSO) alone, offering higher accuracy and faster convergence.[1]

The biggest advantage of using this hybrid method is that it can be used for so many real-world problems, such as finance,

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Table 1: Classification of Algorithm

Numbers	Algorithms	Author(s)	Year of Publication
1	Genetic Algorithm (GA)	Holland	1975
2	Particle Swarm Optimization (PSO)	Kennedy & Eberhart	1995
3	Ant Colony Optimization (ACO)	Dorigo & Gambardella	1997
4	Differential Evolution (DE)	Storn & Price	1997
5	Simulated Annealing (SA)	Kirkpatrick et al.	1983
6	Gravitational Search Algorithm (GSA)	Rashedi et al.	2009
7	Teaching-Learning-Based Optimization (TLBO)	Rao et al.	2011
8	Whale Optimization Algorithm (WOA)	Mirjalili & Lewis	2016

healthcare, engineering, and artificial intelligence. Optimization is important in these fields because it helps improve efficiency and accuracy in decision-making. By combining the strengths of Whale Optimization Algorithm (WOA) or Particle Swarm Optimization (PSO), our approach prevents common problems like being trapped in bad solutions too early. We also discuss why combining two different algorithms is important in optimization research. This research shows that mixing different optimization techniques can create powerful tools for solving complex problems, making them more reliable and useful for a wide range of applications.

1. Proposed Optimization Algorithm

In this research, we select the Whale Optimization Algorithm (WOA) or Particle Swarm Optimization (PSO) hybridization because it delivers the impressive results by effectively balancing exploration and exploitation. This results in higher accuracy and faster convergence than using WOA or PSO individually.

Optimization techniques can be classified into four categories: nature-inspired, evolutionary, human-based, and physics-based algorithms.

1.1 Classification of Algorithm:

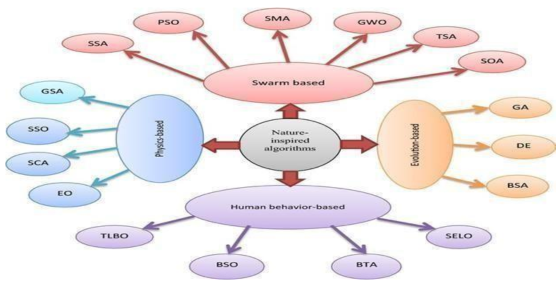

Figure 1: Classification of Algorithm

1.2 Classification of Optimization Techniques Table:

1.3 Flowchart:

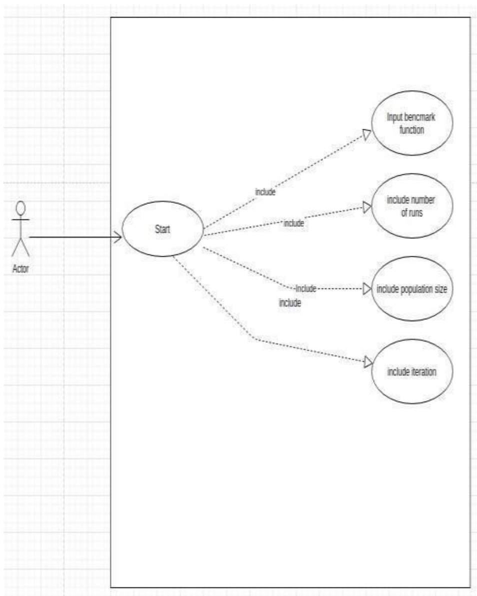

Figure 2: Flow of WOA-PSO [7]

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2.3 Benchmark Functions:

Table 2: Standard UM Benchmark Functions

$F_5(S) = \sum_{m=1}^{z-1}[100(S_{m+1}-S_m^2)^2 + (S_m - 1)^2]$	(10,30,50,100)	[-38 , 38]	0
$F_6(S) = \sum_{m=1}^{z}([S_m + 0.5])^2$	(10,30,50,100)	[-100 , 100]	0
$F_7(S) = \sum_{m=1}^{z} mS_m^4 + random\ [0,1]$	(10,30,50,100)	[-1.28, 1.28]	0

Functions	Dimension	Range	f_{min}
$F_8(S) = \sum_{m=1}^{z} - S_m sin(\sqrt{\lvert S_m \rvert})$	(10,30,50,100)	[-500,500]	-418.98295
$F_9(S) = \sum_{m=1}^{z}[S_m^2 - 10cos\ (2\pi\ S_m) + 10]$	(10,30,50,100)	[-5.12,5.12]	0
$F_{10}(S) = -20exp\ (-0.2\sqrt{\left(\frac{1}{z}\sum_{m=1}^{z} S_m^2\right)}) - exp\ (\frac{1}{z}\sum_{m=1}^{z} cos(2\pi S_m) + 20 + d$	(10,30,50,100)	[-32,32]	0
$F_{11}(S) = 1 + \sum_{m=1}^{z} \frac{S_m^2}{4000} - \Pi_{m=1}^{z} cos\frac{S_m}{\sqrt{m}}$	(10,30,50,100)	[-600, 600]	0
$F_{12}(S) = \frac{\pi}{z}\left\{10\,sin(\pi\tau_1) + \sum_{m=1}^{z-1}(\tau_m - 1)^2[1 + 10sin^2(\pi\tau_{m+1})] + (\tau_z - 1)^2\right\} + \sum_{m=1}^{z} u(S_m, 10,100,4)$ $\tau_m = 1 + \frac{S_m+1}{4}$ $u(S_m, b, x, i) = \begin{cases} x(S_m - b)^i & S_m > b \\ 0 & -b < S_m < b \\ x(-S_m - b)^i & S_m < -b \end{cases}$	(10,30,50,100)	[-50,50]	0
$F_{13}(S) = 0.1\{sin^2(3\pi S_m) + \sum_{m=1}^{z}(S_m - 1)^2[1 + sin^2(3\pi S_m + 1)] + (x_z - 1)^2[1 + sin^2 2\pi S_z)]$	(10,30,50,100)	[-50,50]	0

Functions	Dimensions	Range	f_{min}
$F_{14}(S) = [\frac{1}{500} + \sum_{n=1}^{2} 5\frac{1}{n+\sum_{m=1}^{z}(S_m-b_{mn})6}]^{-1}$	2	[-65.536, 65.536]	1
$F_{15}(S) = \sum_{m=1}^{11}[b_m - \frac{S_1(a_m^2+a_m S_2)}{a_m^2+a_m S_3+S_4}]^2$	4	[-5, 5]	0.00030
$F_{16}(S) = 4S_1^2 - 2.1S_1^4 + \frac{1}{3}S_1^6 + S_1 S_2 - 4S_2^2 + 4S_2^4$	2	[-5, 5]	-1.0316
$F_{17}(S) = (S_2 - \frac{5.1}{4\pi 2}S_1^2 + \frac{5}{\pi}S_1 - 6)^2 + 10(1-\frac{1}{8\pi})cosS_1 + 10$	2	[-5, 5]	0.398
$F_{18}(S) = \left[1+(S_1+S_2+1)^2(19-14S_1+3S_1^2-14S_2+6S_1S_2+3S_2^2)\right] \times \left[30+(2S_1-3S_2)^2(18-32S_1+12S_1^2+48S_2-36S_1S_2+27S_2^2)\right]$	2	[-2,2]	3
$F_{19}(S) = -\sum_{m=1}^{4} d_m\ exp\ (-\sum_{n=1}^{3} S_{mn}(S_m - q_{mn})^2)$	3	[1, 3]	-3.32
$F_{20}(S) = -\sum_{m=1}^{4} d_m\ exp\ (-\sum_{n=1}^{6} S_{mn}(S_m - q_{mn})^2)$	6	[0, 1]	-3.32
$F_{21}(S) = -\sum_{m=1}^{5}[(S - b_m)(S - b_m)^T + d_m]^{-1}$	4	[0,10]	-10.1532
$F_{22}(S) = -\sum_{m=1}^{7}[(S - b_m)(S - b_m)^T + d_m]^{-1}$	4	[0, 10]	-10.4028
$F_{23}(S) = -\sum_{m=1}^{7}[(S - b_m)(S - b_m)^T + d_m]^{-1}$	4	[0, 10]	-10.5363

Functions	Dimensions	Range	f_{min}
$F_1(S) = \sum_{m=1}^{z} S_m^2$	(10,30,50,100)	[-100 , 100]	0
$F_2(S) = \sum_{m=1}^{z} \lvert S_m \rvert + \prod_{m=1}^{z} \lvert S_m \rvert$	(10,30,50,100)	[-10 ,10]	0
$F_3(S) = \sum_{m=1}^{z} (\sum_{n-1}^{m} S_n)^2$	(10,30,50,100)	[-100 , 100]	0
$F_4(S) = max_m\{\lvert S_m \rvert, 1 \le m \le z\}$	(10,30,50,100)	[-100 , 100]	0

3. Results and Discussion

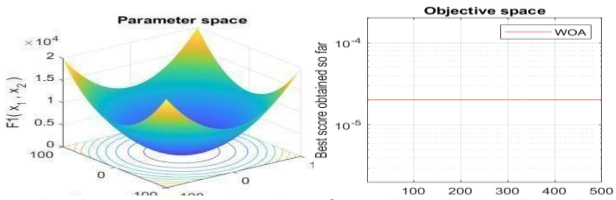
In the below table it shows the comparison between the Whale Optimization Algorithm (WOA) and its Hybrid. the Hybrid approach was tested 23 times and performed better than the Whale Optimization Algorithm (WOA) in most cases. Specifically, the Hybrid approach achieved better results in 14 out of 23 functions. the Hybrid method found significantly lower values, indicating better optimization**.**

Table 3: Original and Hybrid Value

Function	Value of WOA	Value of Hybrid
F1	3.20E-73	0.0043488
F2	1.72E-50	0.0080922
F3	38874.0909	0.074816
F4	65.2387	0.1938
F5	27.9213	0
F6	0.097687	0.0016877
F7	0.0033129	0.0027857
F8	-1.12E+04	-418.9665
F9	0	2.03E-03
F10	4.00E-15	0.21375
F11	0	0.00352
F12	1.13E-02	0.017512
F13	0.43439	0.00016603
F14	0.998	0.998
F15	0.00066602	0.0006734
F16	-1.0316	-1.0645
F17	0.39792	0.34256
F18	3	2
F19	-3.8618	-1.8996
F20	-2.8404	-1.1698
F21	-10.1421	-10.1398
F22	-10.3973	-10.2684
F23	-5.1284	-10.515

Overall, these findings indicate that the Hybrid approach enhances optimization performance, making it a more effective solution in many cases.

Function No.1



The best optimal value found by hybridized algorithm of WOA with PSO was 0.0043488.

Function No.2

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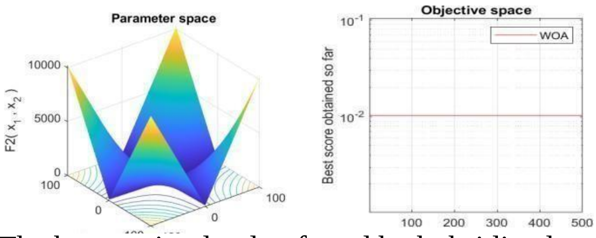
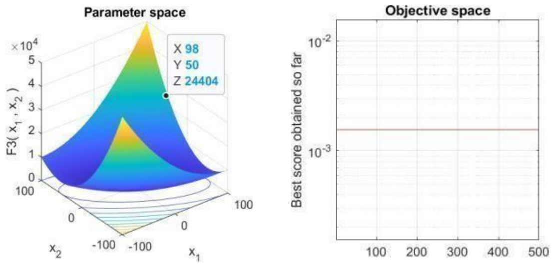


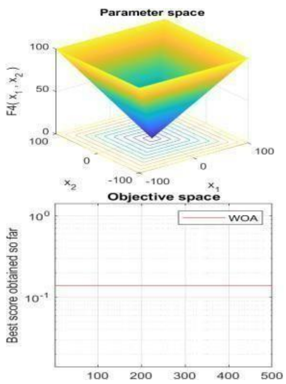
The best optimal value found by hybridized algorithm of WOA with PSO was 0.0080922.

Function No.3



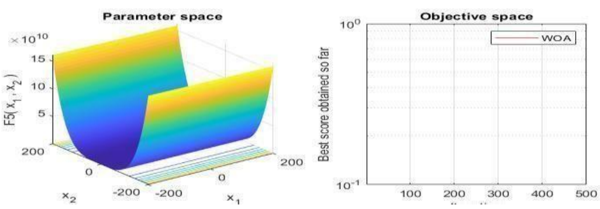
The best optimal value found by hybridized algorithm of WOA with PSO was 0.074816.

Function No.4



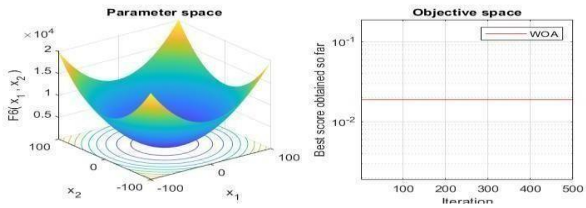
The best optimal value found by hybridized algorithm of WOA with PSO was 0.1938.

Function No.5



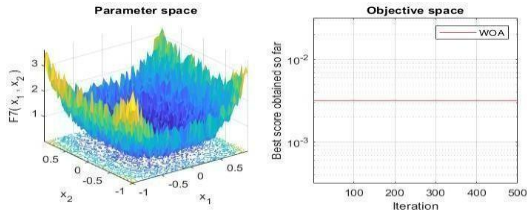
The best optimal value found by hybridized algorithm of WOA with PSO was 0.

Function No.6



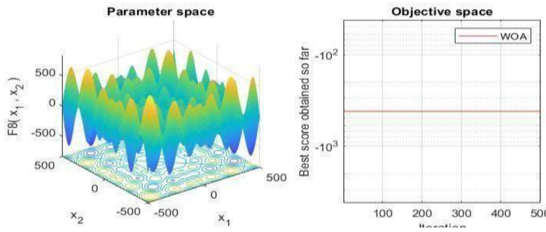
The best optimal value found by hybridized algorithm of WOA with PSO was 0.0016877.

Function No.7



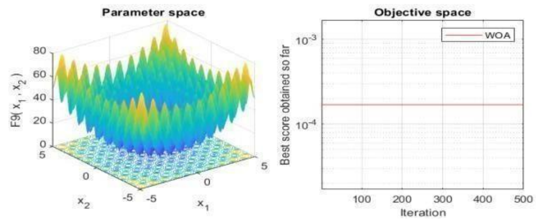
The best optimal value found by hybridized algorithm of WOA with PSO was 0.0027857.

Function No.8



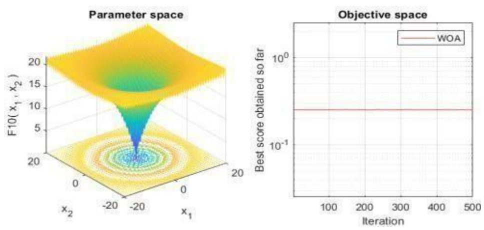
The best optimal value found by hybridized algorithm of WOA with PSO was 0.0027857.

Function No.9



The best optimal value found by hybridized algorithm of WOA with PSO was 2.03E-03.

Function No.10



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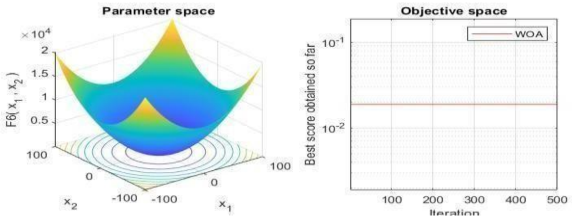
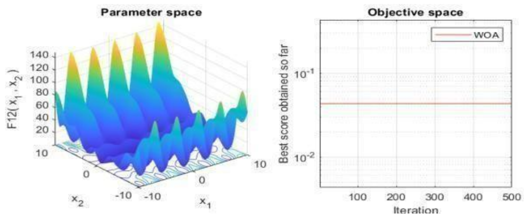
The best optimal value found by hybridized algorithm of WOA with PSO was 0.21375.

Function No.11



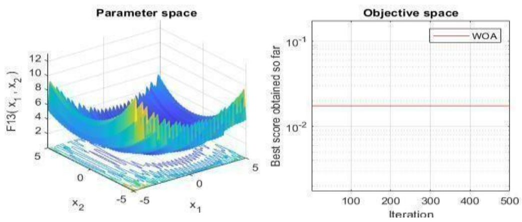
The best optimal value found by hybridized algorithm of WOA with PSO was 0.00352.

Function No.12



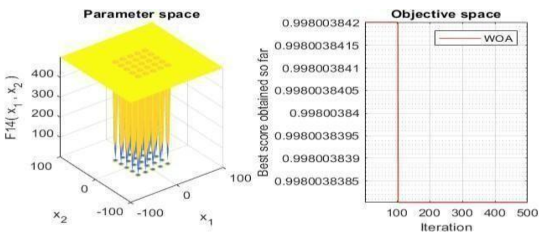
The best optimal value found by hybridized algorithm of WOA with PSO was 0.017512.

Function No.13



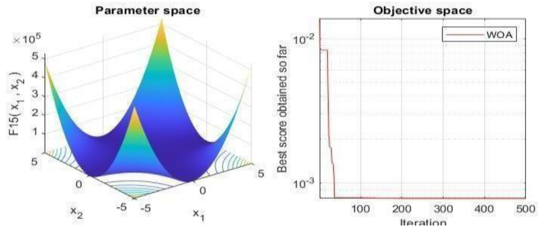
The best optimal value found by hybridized algorithm of WOA with PSO was 0.00016603.

Function No.14



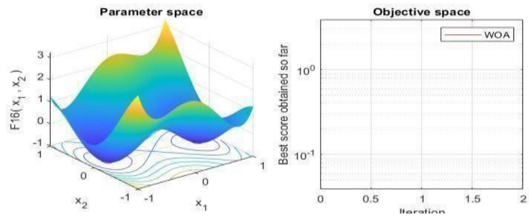
The best optimal value found by hybridized algorithm of WOA with PSO was 0.998.

Function No.15



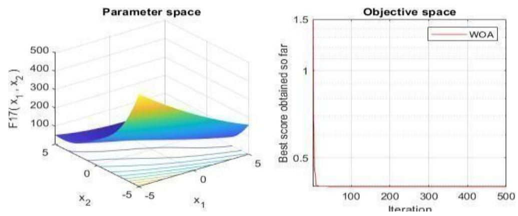
The best optimal value found by hybridized algorithm of WOA with PSO was 0.0006734.

Function No.16



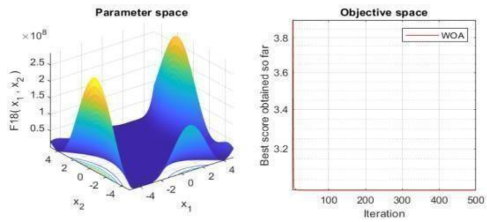
The best optimal value found by hybridized algorithm of WOA with PSO was -1.0645.

Function No.17



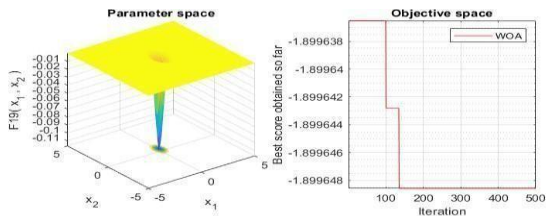
The best optimal value found by hybridized algorithm of WOA with PSO was 0.34256.

Function No.18



The best optimal value found by hybridized algorithm of WOA with PSO was 2.

Function No.19



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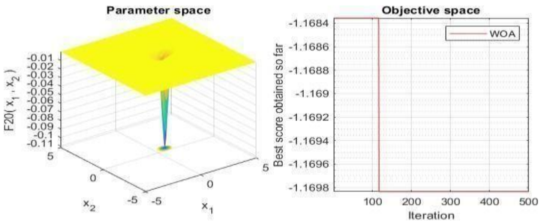
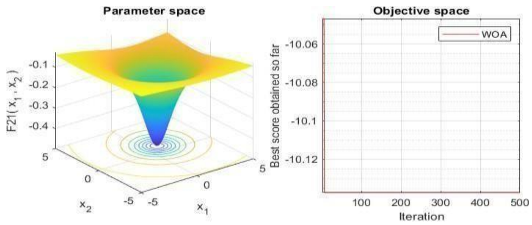
The best optimal value found by hybridized algorithm of WOA with PSO was -1.8996.

Function No.20



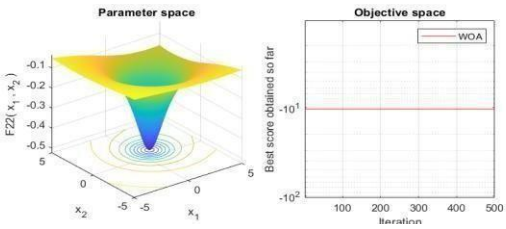
The best optimal value found by hybridized algorithm of WOA with PSO -1.1698.

Function No.21



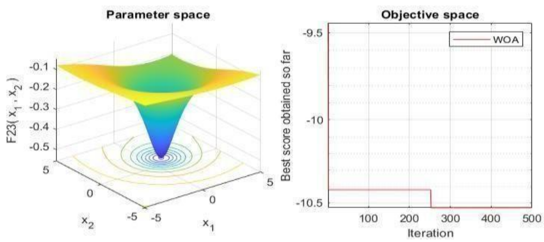
The best optimal value found by hybridized algorithm of WOA with PSO was -10.1398.

Function No.22



The best optimal value found by hybridized algorithm of WOA with PSO was -10.2684.

Function No.23



The best optimal value found by hybridized algorithm of WOA with PSO was -10.515.

8 Conclusion

In conclusion, the hybridized algorithm was tested 23 times and performed better than the Whale Optimization Algorithm (WOA) method in 14 functions which includes F3, F4, F5, F6, F7, F8, F13, F16, F17, F18, F19, F20, F21 and F22. This indicates that the hybrid approach provides better optimization results in most of the function. This means it gave better results for those functions and showing that the hybrid approach's effectiveness in optimization in many cases.
This proves that combining the two algorithms enhance the optimization process more effective and reliable.

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