GWO-HHO Hybrid: Strengthening Grey Wolf Optimizer with Harris Hawks Strategy for Numerical Optimization

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Abstract
Optimization algorithms play a crucial role in solving complex numerical problems across diverse domains. This paper presents a hybrid Grey Wolf Optimizer (GWO) and Harris Hawks Optimization (HHO) algorithm, designed to improve solution accuracy and convergence efficiency. The proposed hybrid approach leverages GWO's structured leadership-based exploration with HHO's dynamic and adaptive hunting strategies, ensuring a balanced trade- off between exploration and exploitation. The performance of the hybrid GWO-HHO algorithm is evaluated on twenty-three benchmark functions, and its results are compared with the original GWO. It is observed that the proposed hybrid approach achieves higher accuracy and improved optimization efficiency. In this paper GWO algorithm is combined with HHO algorithm for numerical optimization.

Keywords:
GWO-HHO, Hybrid, Optimization, Exploration, Exploitation

I. Introduction

Meta-heuristic optimization techniques have gained significant attention due to their ability to solve complex numerical and real-world problems. Among these, the Grey Wolf Optimizer (GWO), inspired by the leadership and hunting strategies of grey wolves, has been widely used for its simplicity and efficiency in maintaining a balance between exploration and exploitation [1]. However, GWO faces challenges such as slow convergence and premature stagnation in local optima. To address these issues, Harris Hawks Optimization (HHO), inspired by the adaptive transition strategies, makes it a strong pack for hybridization with GWO [2]. The integration of GWO and HHO aims to leverage GWO's structured leadership-based search mechanism with aggressive exploration and adaptive strategies, ensuring a more balanced approach to global and local search. This hybridization enhances diversity in the search process, reduces the risk of premature convergence, and improves convergence speed, making it suitable for solving high dimensional and multi-objective optimization problems [4]. The proposed GWO-HHO hybrid algorithm is evaluated on twenty- three benchmark functions, and the results showed improved performance.

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ii. Literature Review

Nature-based algorithms mimic natural processes such as animal behaviour or ecological systems. Evolutionary-based algorithms evolve a population of solutions using selection, crossover, and mutation. Physics-based algorithms leverage physical laws to explore search spaces effectively. Human-based algorithms are inspired by human learning, decision-making, and social behaviours.

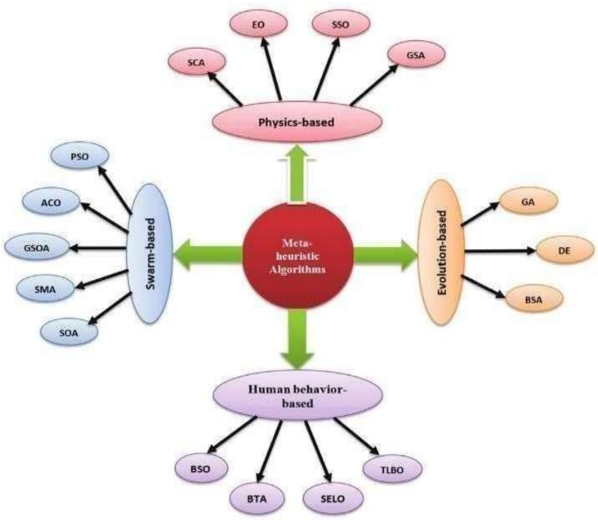

Fig1: Classification of Meta heuristic Algorithms

Sr No.	Algorithm	Author Name	Publication Year
1	Ant Colony Optimization (ACO)	Dorigo & Gambardella	1997
2	Firefly Algorithm (FA)	Xin-She Yang	2008
3	Genetic Algorithm (GA)	John Holland	1975
4	Differential Evolution (DE)	Rainer Storn & Kenneth Price	1995
5	Simulated Annealing (SA)	Scott Kirkpatric k, C. D. Gelatt, M. P. Vecchi	1983
6	Harmony Search(HS)	ZongWoo Geem, Joong Hoon Kim & G.V. Loganathan	2001
7	Exchange Market Algorithm (EMA)	Ali Asgharpoor & Amir Hossein Moosavi Tabatabaei	2014
8	Tabu Search (TS)	Fred W. Glover	1986

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Table1:

For Each Search AgentUpdate The Position

Table1: Metaheuristic Algorithms

1. Pseudo Code

Initialize The Grey Wolf Population Xi (I = 1, 2, ..., N) Initialize A, A, And C
Calculate The Fitness Of Each Search Agent Xα=The Best Search Agent
Xβ=The Second Best Search Agent Xδ=The Third Best Search Agent **While** (T Max Number Of Iterations) (3.7)
OThe Current Search Agent By Equation
End For
Update A, A, And C
Calculate The Fitness Of All Search Agents Update Xα, Xβ, And Xδ
T+1
End While
Return Xα

2. Benchmark Functions

Benchmark Functions Are Crucial In Evaluating Optimization Algorithms By Testing Their Ability To Find The Global Minimum In Complex Landscapes. These Functions Range From Simple Convex Ones Like Sphere To Highly Multimodal And Deceptive Ones Like Rastrigin And Schwefel. They Help Measure The Convergence Speed, Accuracy, And Robustness Of Algorithms Like The Grey Wolf Optimizer (GWO). Below Is A Brief Explanation Of The Twenty- Three Benchmark Functions Used In GWO, Along With Their Mathematical Equations.

Table 2: Standard UM benchmark functions

Functions	Dimensions	Range	f_{min}
$F_1(S)=\sum_{m=1}^{z} S_m^2$	(10,30,50,100)	[-100, 100]	0
$F_2(S)=\sum_{m=1}^{z}\lvert S_m\rvert+\prod_{m=1}^{z}\lvert S_m\rvert$	(10,30,50,100)	[-10,10]	0
$F_3(S)=\sum_{m=1}^{z}(\sum_{n=1}^{m} S_n)^2$	(10,30,50,100)	[-100, 100]	0
$F_4(S)=max_m\{\lvert S_m\rvert, 1\leq m\leq z\}$	(10,30,50,100)	[-100, 100]	0
$F_5(S)=\sum_{m=1}^{z-1}[100(S_{m+1}-S_m^2)^2+(S_m-1)^2]$	(10,30,50,100)	[-38, 38]	0
$F_6(S)=\sum_{m=1}^{z}([S_m+0.5])^2$	(10,30,50,100)	[-100, 100]	0
$F_7(S)=\sum_{m=1}^{z} mS_m^4+random[0,1]$	(10,30,50,100)	[-1.28, 1.28]	0

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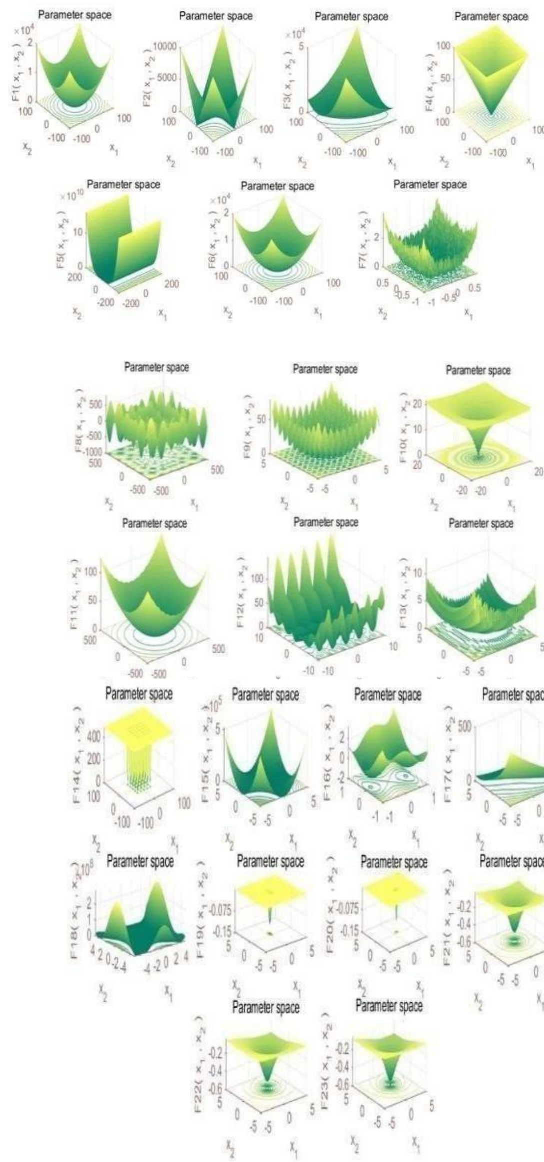
$F_8(S) = \sum_{m=1}^{z} -S_m sin(\sqrt{\|S_m\|})$	(10,30,50,100)	[-500,500]	-418.98295
$F_9(S) = \sum_{m=1}^{z} [S_m^2 - 10cos(2\pi S_m) + 10]$	(10,30,50,100)	[-5.12,5.12]	0
$F_{10}(S) = -20exp(-0.2\sqrt{\left(\frac{1}{z}\sum_{m=1}^{z} S_m^2\right)}) - exp(\frac{1}{z}\sum_{m=1}^{z} cos(2\pi S_m) + 20 + d$	(10,30,50,100)	[-32,32]	0
$F_{11}(S) = 1 + \sum_{m=1}^{z} \frac{S_m^2}{4000} - \Pi_{m=1}^{z} cos\frac{S_m}{\sqrt{m}}$	(10,30,50,100)	[-600, 600]	0
$F_{12}(S) = \frac{\pi}{z}\left\{10\sin(\pi\tau_1) + \sum_{m=1}^{z-1}(\tau_m - 1)^2[1 + 10sin^2(\pi\tau_{m+1})] + (\tau_z - 1)^2\right\} + \sum_{m=1}^{z} u(S_m, 10,100,4)$ $\tau_m = 1 + \frac{S_m+1}{4}$ $u(S_m, b, x, i) = \begin{cases} x(S_m - b)^i & S_m > b \\ 0 & -b < S_m < b \\ x(-S_m - b)^i & S_m < -b \end{cases}$	(10,30,50,100)	[-50,50]	0
$F_{13}(S) = 0.1\{sin^2(3\pi S_m) + \sum_{m=1}^{z}(S_m - 1)^2[1 + sin^2(3\pi S_m + 1)] + (x_z - 1)^2[1 + sin^2 2\pi S_z)]$	(10,30,50,100)	[-50,50]	0
$F_{14}(S) = [\frac{1}{500} + \sum_{n=1}^{2} 5\frac{1}{n + \sum_{m=1}^{z}(S_m - b_{mn})6}]^{-1}$	2	[-65.536, 65.536]	1
$F_{15}(S) = \sum_{m=1}^{11}[b_m - \frac{S_1(a_m^2 + a_m S_2)}{a_m^2 + a_m S_3 + S_4}]^2$	4	[-5, 5]	0.00030
$F_{16}(S) = 4S_1^2 - 2.1S_1^4 + \frac{1}{3}S_1^6 + S_1S_2 - 4S_2^2 + 4S_2^4$	2	[-5, 5]	-1.0316
$F_{17}(S) = (S_2 - \frac{5.1}{4\pi 2}S_1^2 + \frac{5}{\pi}S_1 - 6)^2 + 10(1 - \frac{1}{8\pi})cosS_1 + 10$	2	[-5, 5]	0.398
$F_{18}(S) = \left[1 + (S_1 + S_2 + 1)^2(19 - 14S_1 + 3S_1^2 - 14S_2 + 6S_1S_2 + 3S_2^2)\right] \times \left[30 + (2S_1 - 3S_2)^2(18 - 32S_1 + 12S_1^2 + 48S_2 - 36S_1S_2 + 27S_2^2)\right]$	2	[-2,2]	3
$F_{19}(S) = -\sum_{m=1}^{4} d_m exp(-\sum_{n=1}^{3} S_{mn}(S_m - q_{mn})^2)$	3	[1, 3]	-3.32
$F_{20}(S) = -\sum_{m=1}^{4} d_m exp(-\sum_{n=1}^{6} S_{mn}(S_m - q_{mn})^2)$	6	[0, 1]	-3.32
$F_{21}(S) = -\sum_{m=1}^{5}[(S - b_m)(S - b_m)^T + d_m]^{-1}$	4	[0,10]	-10.1532
$F_{22}(S) = -\sum_{m=1}^{7}[(S - b_m)(S - b_m)^T + d_m]^{-1}$	4	[0, 10]	-10.4028
$F_{23}(S) = -\sum_{m=1}^{7}[(S - b_m)(S - b_m)^T + d_m]^{-1}$	4	[0, 10]	-10.5363

3. Search Space

The search space is the range of possible solutions in an optimization problem, bounded by upper and lower limits. A larger space allows better exploration but increases complexity, while a smaller one speeds up convergence but may miss the optimal solution. Efficient algorithms balance both for optimal results.



4. Results & Discussions

The proposed hybrid GWO-HHO algorithm demonstrates significant improvements over the original GWO algorithm in numerical optimization. Out of the twenty-three benchmark functions tested, enhancements were observed in fourteen cases (1,2,3,4,5,6,8,10,11,15,20,21,22,23),

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Functions	Original Value	Hybrid Value
F1	6.18E-28	1.0754E-21
F2	2.98E-16	2.77E-13
F3	6.23E-06	1.88E-01
F4	5.92E-07	3.33E-02
F5	27.0188	26.5475
F6	1.2563	0.75304
F7	0.0013123	0.0044082
F8	-6344.7997	-5417.6748
F9	5.68E-14	1.83E+01
F10	1.46E-13	1.38E-12
F11	0.011816	0
F12	0.021455	0.035293
F13	0.5825	0.69339
F14	0.998	0.998
F15	0.00041885	0.00030755
F16	-1.0316	-1.0316
F17	0.39789	0.39789
F18	3.0001	3
F19	-3.8557	-3.8626
F20	-3.322	-3.196
F21	-10.1512	-5.0552
F22	-10.4014	-5.0877
F23	-10.5333	-5.1284

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showcasingbetter optimization performance. While some values remained unchanged and others exhibited fluctuations, overall results highlight the effectiveness of the hybrid approach in achieving more optimal and precise solutions. The following analysis
further explores these findings in detail.

Conclusion

This research improves the performance ofthe Grey Wolf Optimization Algorithm using the Hybridization approachof (GWO+HHO). Out of twenty-three benchmark functions, fourteen functions achieved better optimal values compared to the original one, demonstrating an improvement in GWO's performance

References

S. Mirjalili, S.M. Mirjalili and A. Lewis, "Grey wolf optimizer," Advances in Engineering Software, vol. 69, pp. 4661, 2014, doi: 10.1016/j.advengsoft.2013.12.007

A. A. Heidari et al., "Harris hawks **optimizationAlgorithm and applicat**

X. Wang, T.M. Choi, H Liu, and X. Yue, "A novelhybrid optimization algorithm for emergency transportation problems," IEEE Transactions on Systems, Man, and Cybernetics: Systems, 2018, doi: 10.1109/TSMC.2016.2606440.

R. V. Rao and G. G. Waghmare, "A new optimization algorithm for solving complex constrained design optimization problems," vol. 0273, no. April, 2016, doi: 10.1080/0305215X.2016.1164855.
R. V. Rao and G. G. Waghmare, "A new optimization algorithm for solving complex constrained design optimization problems," vol. 0273, no. April, 2016, doi: 10.1080/0305215X.2016.1164855.

D. Yousri, T. S. Babu, and A. Fathy, "Recent methodology based Harris hawks optimizer for designing load frequency control incorporated in multi- interconnected renewable energy plants," Sustain. Energy, Grid Networks, 2020, doi: 10.1016/j.segan.2020.100352.

Y. Cheng, S. Zhao, B. Cheng, S. Hou, Y. Shi, and J. Chen, "Modeling and optimization for collaborative business process towards IoT applications," Mob. Inf. Syst., 2018, doi: 10.1155/2018/9174568.